# Probability Rules

**1.** Sum of P(ALL possible outcomes) = 1
**2. Complement of an event**: Probability that event A does not occur, P(A') = 1 – P(A)
E.g. If P(A) = 5% = 0.05, then P(A') = 95% = 0.95
**Caution:** 0.05 is <u>not</u> the same as 0.05%! Do <u>not</u> mix–up percentages and decimals. You need use ONLY one of those formats!
**3. Mutually Exclusive or Disjoint events** are those that CANNOT occur simultaneously.
If A and B are mutually exclusive,
a. For Mutually Exclusive Events, P(A *and* B) = ZERO
b. For Mutually Exclusive Events, P(A *or* B) = P(A) + P(B) <––––––– understand & memorize this.

**DEMONSTRATION:**
P(you wear a green shirt ~ Event A) = 30%,
P(you wear a yellow shirt ~ Event B) = 50%
Assuming that you shant wear BOTH shirts together [!],
P(you wear a green shirt AND yellow shirt) i.e. P(A and B) = 0 while
P(you wear a green shirt OR yellow shirt) i.e. P(A or B) = 30% + 50% = 80%

**Use the OR Rule of Addition *only* for Mutually Exclusive events.**

**CONCEPT** How do you know the outcomes are Mutually Exclusive? Well, we are usually talking about a SINGLE trial or experiment [NO repetitions]. For the *same* "trial" or experiment, the different outcomes are Mutually Exclusive.
E.g. **A.** for a shopper purchasing a car, the the *different* car–purchase outcomes [Japanese, American, German, French] are Mutually Exclusive! If you're buying *one* car, it is EITHER Japanese OR American OR German OR French, etc. A car is not Japanese AND American AND German AND French.
But the key thing to note is: we're talking about ONE shopper or ONE car!
**B.** The *different* outcomes of *one* throw of an icosahedral [20–sided] die [1...20] are Mutually Exclusive. The key thing to note is: we're talking about ONE die.
**C.** The *different* outcomes of *one* spin of a roulette wheel [00...36] are Mutually Exclusive. the key thing to note is: we're talking about ONE spin.

**4. Independent Events** are those whose outcomes don't affect / influence one another. **<–––––––––– memorize this.**
If A and B are independent events,
P(A occurs, *given* that B has occurred) = P(A)...because B has NO influence on A, so who cares if B has occurred! **<–––––––––– understand this well.**
P(B occurs, *given* that A has occurred) = P(B)...because A has NO influence on B, so who cares if A has occurred! **<–––––––––– understand this well.**

**For Independent Events, P(A and B) = P(A) · P(B) <––––––––––––– memorize this.**
**Use the AND Rule of Multiplication *only* for Independent events.**

**DEMONSTRATION:** Suppose 2 dice are thrown. Clearly the outcomes are independent of each other. Also, there are 36 possible outcomes:

Lets see if the above Multiplication Rule holds!
P(Die 1 shows a 6 *in general*) = 1/6 [or, from the Dice Table above: 6/36]
P(Die 2 shows a 4 *in general*) = 1/6 [or, from the Dice Table above: 6/36]
P(Die 1 shows a 6 AND Die 2 shows a 4) = 1/36 [Look at the table!]
And: 1/6 · 1/6 = 1/36!

**One of the most common sources of confusion is between Mutually Exclusive Events and Independent events.**

**CONCEPT** It is only when a "trial" or an experiment is *repeated* e.g. 2 dice are rolled, 10 coins are flipped, 100 shoppers are polled, 15 bottles (prize vs. no prize) are opened, 30 cars are purchased (foreign or U.S. brand), a roulette wheel is spun 100 times, etc. that it makes sense to talk about Independent Events since the outcomes or choices or preferences are INDEPENDENT of each other! **Read this AGAIN.**

**CONTRAST** For Mutually Exclusive Events, we're talking about the different outcomes of ONE trial which *cannot* occur together [coin toss H–T, die roll 1–2–3–4–5–6, weather rain–not rain]! For Independent Events, we're talking about REPEATED trials [5 coin tosses, 10 dice rolls, weather for 30 days]. **Read this AGAIN.**

**4. General OR Rule for Non–Mutually Exclusive Events** i.e. Events that [can] *occur* simultaneously
P(A or B) = P(A) + P(B) – P(A and B) **<––––––––––––––––– memorize this.**
[since the middle area ~ P(A and B) ~ the Intersection of A and B, would be counted twice when we add P(A) and P(B)]

**5. Calculating P(At Least One Success)**
Suppose P(taking No Calculus) = 0.05,
P(taking 1 Calculus class) = 0.1,
P(taking 2 Calculus classes) = 0.2,
P(taking 3 calculus classes) = 0.4,
P(taking 4 Calculus classes) = 0.25

**Problem.** What is the probability that a student takes ***at least*** 1 Calculus class?
**SOLUTION:**
**(LONG WAY)** Required P( ≥ 1 Calculus) = P(taking 1 Calculus class) + P(taking 2 Calculus classes) + P(taking 3 calculus classes) + P(taking 4 Calculus classes)
= 0.1 + 0.2 + 0.4 + 0.25 **= 0.95**
**(SMART WAY)** Required P( ≥ 1 Calculus) = 1 – P(taking No calculus)
= 1 – 0.05 = **0.95**

**General Concept:**
P(at least 1 occurrence of event) = 1 – P(event does not occur at all) **<–––––––– memorize this.**

Stated differently [this is a better formulation!], suppose there are *n* trials / repetitions. Then,
**P(at least 1 occurrence of an Event on *n* trials)**
**= 1 – P(Event does *not* occur on *each* of the *n* trials) <––––––– understand & memorize this.**

**Explanation:** Since P(event does *not* occur in <u>any</u> of the $n$ trials) + P(event occurs <u>at least once</u> in $n$ trials) = 100%

**Subtracting:**
=> P(event occurs <u>at least once</u> in $n$ trials) = 1 – P(event does *not* occur in any of the $n$ trials)
**This is the At–Least One Rule Short–cut.**

**Problem.** 1. Suppose there's a 53% likelihood that a child born is Male. If a family has 4 children, what is the probability that at least 1 shall be a boy?
2. Suppose a team has a 26% chance of winning a game. If the team plays 6 games, what is the probability that it shall win at least 1 game?
3. Suppose 41% of voters are Democrats. If 3 voters are randomly selected, what is the probability that at least 1 shall be a Democrat?

**Solution.**
1. P(B) = 0.53
P(at least 1 boy amongst 4 children)
= 1 – P(no boy at all amongst the 4 children)
= 1 – P(B', B', B', B')
= 1 – $P(B')^4$, assuming independence (so we multiply probabilities)
= 1 – $0.47^4$
2. P(W) = 0.26
P(at least 1 win amongst 6 games)
= 1 – P(no win at all amongst the 6 games)
= 1 – P(W', W', W', W', W', W')
= 1 – $P(W')^6$, assuming independence (so we multiply probabilities)
= 1 – $0.74^6$
3. P(D) = 0.41
P(at least 1 Democrat amongst 3 voters)
= 1 – P(no Democrat at all amongst the 3 voters)
= 1 – P(D', D', D')
= 1 – $P(D')^3$, assuming independence (so we multiply probabilities)
= 1 – $0.59^3$

**6. The concept of a "trial" is broad: any *repetition* is a trial in statistics.**

- Pick 10 voters ~ 10 trials.
- Select 15 shoppers ~ 15 trials.
- Roll a die 7 times ~ 7 trials.
- Roll 7 dice all at once ~ 7 trials.
- Play a game 60 times ~ 60 trials.
- Go to school 5 times a week ~ 5 trials.
- Spin a roulette wheel 6 times ~ 6 trials.

The **Conditional probability** that event A occurs, *given that B has occurred*, is given by: P(A, given B) **=** P(A | B) = P(A and B) / P(B)

This relationship is **always** true. It doesnt matter if the events are Mutually Exclusive or Independent – the problem shall work itself out!

**CAUTION!**

1. The event that is *given* to us – the Q shall clearly INDICATE that! – is written after the | symbol. To remember the formula, treat the event B like a fraction (/ B)...so that it becomes the denominator!

2. For table Qs, for the numerator, simply read off the table. That is, the numerator P(A and B) is simply obtained by looking at the "cell" that is the *intersection* of events A and B.

   **Caution!** Do *not* use the AND Rule for Independent Events: it is applicable only if A and B are known are *given* to be Independent! In 99% of the TABLE situations, the outcomes are *not* independent.

   For the numerator, do ***not*** do: P(A and B) = P(A)·P(B) *unless* it is GIVEN or SUGGESTED that A and B are independent! The denominator, P(B) is simply at the *end* of the relevant row or *bottom* of the relevant column [depending on where B and B' are located – as a row or column?]

Interpret Conditional Probability Qs carefully since it would suggest what gets placed *after* the | in P( A | B). The following constructions usually indicate a conditional event [study these well!]:

**Corollary**: From the conditional probability formula, upon cross–multiplication, we get:
P(A and B) = P(B)·P(A | B) = P(A)·P(B | A)

The general pattern holds:

- P(**A and B**)·P(C | **A and B**) = P(**A and B and C)**
- P(**A and B and C**) · P(D | **A and B and C**) = P(**A and B and C and D)**

and so on.

**Example:** If P(Recent donor) = 0.5, P(Pledge | Recent Donor) = 0.4 and P(Check | Recent Donor and Pledge) = 0.8, then using the pattern above:
P(Recent Donor AND Pledge AND Check) = 0.5·0.4·0.8 = 0.016

Why? Because P(Recent donor)·P(Pledge | Recent Donor) = P(Recent Donor and Pledge) and in turn, P(Recent Donor and Pledge)·P(Check **|** Recent Donor and Pledge**)**
= P(Recent Donor AND Pledge AND Check)

so that multiplying across:

P(Recent donor)·P(Pledge | Recent Donor)·P(Check | Recent Donor and Pledge) = P(Recent donor AND Pledge AND Check)

If the Q does not mention any method, you might use:

- a Probability Table, if applicable **OR**
- a Tree Diagram, if applicable **OR**

- *neither*: you may simply employ probability rules and relationships we've learnt! For instance, P(A and B) / P(A)
  = [P(B)·P(A | B)] / [P(A and B) + P(A and C) + P(A and D)]
  = [P(B)·P(A | B)] / [P(B)·P(A | B) + P(C)·P(A | C) + P(D)·P(A | D)]
  and so on!

**Independent Events:** If A and B are independent, one's outcome does not depend on / affect the others'...so that:

- P(A | B) = P(A)
- P(B | A) = P(B)

**Alternately,** independent events are those whose outcomes don't affect / influence one another. Check for Independence using either of the above bullet–points.

**The General OR Rule:** P(A or B) = P(A) + P(B) – P(A and B)

**Mutually Exclusive Events:** are events that don't / cannot occur simultaneously: P(A and B) = 0. Consequently, P(A *or* B) = P(A) + P(B). This is the **OR Rule of Addition.** Use the OR Rule of Addition *only* for Mutually Exclusive events.

The **AND Rule of Multiplication:** For independent events [*only*!], P(A and B) = P(A)·P(B)
Do *not* use the AND Rule of Multiplication whenever an AND is perceived! It is applicable *only* for Independent events.

For ***non*–Mutually Exclusive events**, P(A or B) can be calculated in 3 ways:
i) The General OR Rule: P(A or B) = P(A) + P(B) – P(A or B)
ii) The At–Least–One Rule: P(A or B) = 1 – P(A' and B')
After all, P(A or B) ~ P(At least 1 of them occurs)
iii) Adding up the relevant "cells" / outcomes from the table:
P(A or B) = P(A and B') + P(A' and B) + P(A' and B')

**Problem.** 55% of melons in a large shipment of fruits are ripe (just right) whereas 27% are over-ripe.
a) What proportion of melons are under–ripe [raw]?
b) Suppose 5 melons are chosen. Assume that the ripeness of melons is independent of each other.
(i) What is the probability that the $2^{nd}$ one is ripe?
(ii) What is the probability that the first 2 are both over–ripe?
(iii) What is the probability that the last 3 melons are under–ripe?
(iv) What is the probability that none of the melons are over–ripe?
(v) What is the probability that the $2^{nd}$ melon is the only ripe melon?
(vi) What is the probability that at least 1 melon is ripe?
(vii) What is the probability that some melons are under–ripe? **Tip!** Some ≈ At Least One

**Solution.**
a) P(R) = 0.55, P(OR) = 0.27
Therefore, P(UR) = 1 – P(R) – P(OR) = 1 – 0.55 – 0.27 = 0.18
b)

(i) TRICK QUESTION! [The other melons are <u>not</u> addressed, so simply!] P(R) = 0.55

(ii) TRICK QUESTION! [The other melons are <u>not</u> addressed, so simply!] P(OR, OR) = $P(OR)^2$, assuming independence = $0.27^2$

(iii) TRICK QUESTION! [The other melons are <u>not</u> addressed, so simply!] P(UR, UR, UR) = $P(UR)^3$, assuming independence = $0.18^3$

(iv) P(OR', OR', OR', OR', OR') = $P(OR')^5$, assuming independence = $0.73^5$

(v) **NOTE**: It's obvious that we know what kind of melons the others are!

P(R', R, R', R', R') = $P(R)\cdot P(R')^4$, assuming independence

= $0.55*0.45^4$

(vi) Using the **AT–LEAST 1 RULE** [see Notes above!]:

P(At least 1 ripe melon) = P(at least 1 R)

= 1 – P(None of the melons are Ripe)

= 1 – P(R', R', R', R', R')

= 1 – $P(R')^5$, assuming independence

= 1 – $0.45^5$

(vii) Using the **AT–LEAST 1 RULE** [see Notes above!]:

P(some Under–ripe melons) ~ P(At least 1 Under–ripe melon) **Think about it!**

= P(at least 1 UR)

= 1 – P(None of the melons are Under–Ripe)

= 1 – P(UR', UR', UR', UR', UR')

= 1 – $P(UR')^5$, assuming independence

= 1 – $0.82^5$

**Problem.** The probability that one can sing is 0.34, that one can dance is 0.43, and one can sing or dance is 0.75.

a) Use Notation to describe the probabilities.

b) Find the probability that a randomly chosen person can sing and dance. Use Notation to describe the question and solve.

c) What is the probability that someone can sing but not dance? **Tip!** Make a 2X2 table $1^{st}$!

d) What is the probability that someone can only sing or only dance [but not both]?

e) Are dancing and singing independent of each other? Explain using numerical evidence.

## Solution.

a) P(S) = 0.34, P(D) = 0.43 and P(S or D) = 0.75

b) P(S or D) = P(S) + P(D) – P(S and D) so that 0.75 = 0.34 + 0.43 – x and solving: x = 0.02

c) P(S and D') = 0.32

| | D | D' | |
|---|---|---|---|
| S | 0.02 | 0.32 | 0.34 |
| S' | 0.41 | 0.25 | 0.66 |
| | 0.43 | 0.57 | 1 |

d) P(only S or only D) = P(S and D') + P(S' and D) = 0.32 + 0.41 = 0.73
e) If S and D were independent of each other, then P(S and D) = P(S)·P(D)
But LHS: 0.02 ≠ RHS = 0.34·0.43 = 0.1462
Therefore, S and D are not independent.

**Problem.** A fellow owns a washing machine, a blender, and a dishwasher. The probability that they are working are, respectively: 70%, 95%, 98%. What is the probability that
a) none of them is working?
b) at least 1 of them is working?
c) at least 1 of them is not working?
d) exactly 1 of them is working?

## Solution.

a) P(WM' and B' and DW') = P(WM')·P(B')·P(DW'), assuming independence
**Finish!**
b) P(at least 1 is working)
= 1 – P(none of the 3 is working)
= 1 – P(WM' and B' and DW')
= 1 – P(WM')·P(B')·P(DW'), assuming independence
**Finish!**
c) P(at least 1 is not working) = **LOGIC** this is basically 1 – P(none of them is NOT working)
=1 – P(all are working)
= 1 – P(WM)·P(B)·P(DW), assuming independence
**Finish!**
d) P(exactly 1 is working) = P(B and WM' and DW') + P(B' and WM and DW') + P(B' and WM' and DW) = P(B)·P(WM')·P(DW') + P(B')·P(WM)·P(DW') + P(B')·P(WM')·P(DW), assuming independence
**Finish!**

**Problem.** 44% of Americans have been to an amusement park and 71% to an amusement park or a beach. **You may assume that the 2 events are independent of each other.**
a) Use Notation to describe the probabilities.
b) Find the probability that a randomly selected American has been a beach. Use the given information to work backwards.
c) Find the proportion of Americans that have been to neither. Use Notation to describe the question and solve.
d) What is the probability that an American has been to an amusement park *only*? Use Notation to describe the question and solve.
e) What is the probability that an American has been to an amusement park (only) or the beach (only) but not both? Use Notation carefully to describe the question and solve.

## Solution.

a) P(AP) = 0.44, P(AP or B) = 0.71
b) P(AP or B) = P(AP) + P(B) – P(AP)·P(B), given independence
so that 0.71 = 0.44 + x – 0.44x and solving: x = P(AP and B) = 0.4821 [**use 4 decimals**]
c) P(AP' and B') = P(AP')·P(B') = 0.56·0.5179 = 0.29

d) P(AP and B') **Show steps and Finish.**
e) P(AP' and B) + P(AP and B') **Show steps and Finish.**

**Problem.** Interpret the following Qs in Conditional Probability Notation remembering that the ***given information*** gets placed *after* the | in P( A | B). A conditional event is where we're interested in a SUBSET of the population i.e. the Q deals with a GIVEN narrower set of the population.
a) Amongst the Independent voters, the proportion of Asians was 14%
b) What is the probability that a foreign film was a comedy?
c) The probability that a WalMart customer was unhappy was 23%.
d) Of those who were Hispanic, 33% were married.
e) 96% of Females passed the exam.
f) Suppose a randomly selected student had plagiarized. What is the probability that he / she would be expelled?
g) 71% of items from supplier X were non-Defective.
h) Given that a randomly chosen individual was pulled over, what is the probability that he / she was Black?

## Solution.
a) P(A | I) = 0.14
b) P(C | F) = ?
c) P(U | WM) = 0.23
d) P(M | H) = 0.33
e) P(PE | F) = 0.96
f) P(E | P) = ?
g) P(D' | X) or P(ND | X) = 0.71
h) P(B | PO)

**Problem.** Suppose 45% of balloons in a large bag are Yellow, and you draw 3 balloons at random. Translate the following expressions into algebra and *show* the Multiplication Rule – **when necessary –** to simplify:
**Caution!** All of these are <u>different</u>.
a) P(None are Yellow)
b) P(All are Yellow)
c) P(Only the $1^{st}$ is Yellow)
d) P(The $1^{st}$ is Yellow)
e) P(At least 1 balloon is Yellow)
f) P(Exactly 1 is Yellow)
g) P(the $1^{st}$ Yellow balloon is the $2^{nd}$ selected)
h) P(the $2^{nd}$ balloon is Yellow)
i) P(Only the $2^{nd}$ balloon is Yellow)
j) P(2 balloons are Yellow)
k) P(the $1^{st}$ 2 balloons are Yellow)
l) P(the $3^{rd}$ balloon is Yellow)

m) P(Only the $3^{rd}$ balloon is not Yellow)

n) P(only the $1^{st}$ 2 balloons are Yellow)

## Solution.

a) P(None Yellow) = $P(Y')^3$, assuming independence

b) P(All are Yellow) = $P(Y)^3$, assuming independence

c) P(Only the $1^{st}$ is Yellow) = P(Y, Y', Y') = $P(Y)\cdot P(Y')^2$, assuming independence

d) P(The $1^{st}$ is Yellow) = P(Y) since we dont care about the rest...

e) P(at least 1 Y) = 1 – P(Y', Y', Y') = 1 – $P(Y')^3$, assuming independence

f) P(Exactly 1 is Yellow) = $3\cdot P(Y)\cdot P(Y')^2$, assuming independence since there are 3 possibilities whose probabilities are the same: P(Y, Y', Y'); P(Y', Y, Y') and P(Y', Y', Y)

g) P(the $1^{st}$ Yellow is the $2^{nd}$ balloon selected) ~ P(Y', Y)= $P(Y')\cdot P(Y)$, assuming independence

h) P(the $2^{nd}$ balloon is Yellow) = P(Y) since we dont care about the rest...

i) P(Only the $2^{nd}$ balloon is Yellow) ~ P(Y', Y, Y') = $P(Y')^2\cdot P(Y)$, assuming independence

j) P(2 balloons are Yellow) = $3\cdot P(Y')\cdot P(Y)^2$, assuming independence since there are 3 possibilities: P(Y, Y, Y'); P(Y', Y, Y) and P(Y, Y', Y)

k) P(the $1^{st}$ 2 balloons are Yellow) = $P(Y)^2$, assuming independence , since we dont care about the $3^{rd}$

l) P(the $3^{rd}$ balloon is Yellow) = P(Y)

m) P(Only the $3^{rd}$ balloon is not Yellow) ~ P(Y, Y, Y') = $3\cdot P(Y')\cdot P(Y)^2$, assuming independence

n) P(Only the $1^{st}$ 2 balloons are Yellow) ~ P(Y, Y, Y') = $P(Y)^2\cdot P(Y')$

**Problem.** In an election, 52% of voters were women; 45% of voters voted Republican; and 38% were women and Republican.

**1.** Construct and fill a **Probability Table** first to illustrate the situation.

**2.** Determine the probability that a voter was a <u>woman</u> but <u>not a Republican</u>.

**3.** Determine the probability that a voter was a woman <u>if it is known that she was a Republican</u>. **<––––––––––––––– GIVEN INFORMATION.**

**4.** Determine the probability that a voter was not a woman <u>given that the voter was not Republican</u>. **<––––––––––––––– GIVEN INFORMATION.**

**5.** Determine the probability that a voter was neither a woman nor a Republican.

**6.** Determine the probability that a voter was male and a Republican.

**7.** Calculate the probability that a voter was a woman or a Republican.

**8.** Are the events Republican and Woman *disjoint* (ie. *mutually exclusive*)? Explain.

**9.** What is the probability <u>that a voter who is a woman</u> is a Republican.

**10.** What is the probability that a randomly chosen individual would be a Republican or a woman but not both?

**11.** Are the factors Republican and Woman independent of each other?

## Solution.

**1.**

| | Republican | Republican' | Total |
|---|---|---|---|
| **Woman** | 0.38 | 0.14 | 0.52 |
| **Woman'** | 0.07 | 0.41 | 0.48 |
| **Total** | 0.45 | 0.55 | 1 |

**2.** P(W and R') = 14% [simply reading off the table!]
**3.** P(W, given R) = P(W and R) / P(R)
= 0.38/0.45
**Explanation:** We are *given* that the voter was Republican. So we need to constrain our subset to 0.45 voters only! Out of those, 0.38 were women...
**4.** P(W', given R') = P(W' and R')/P(R')
= 0.41/0.55
**Explanation:** As before, we are *given* that the voter was Not a Republican. So we need to constrain our subset to 0.55 voters only! Out of those, 0.41 were *not* women...
**5.** Directly (!), P(W' and R') = 0.41
**6.** Directly (!), P(W' and R) = 0.07
**7.** As explained, there are 3 ways to do the 2–event OR question: **you <u>need</u> to know all 3 approaches!**
**Method 1** P(W or R) = P(W) + P(R) – P(W and R)...**using the General Or Rule**
= 0.52 + 0.45 – 0.38
= 0.59
**Method 2** P(W or R) = P(W and R') + P(W' and R) + P(W and R)...**using common sense to examine which of the interior 4 cells satisfy the W or R condition!**
= 0.14 + 0.07 + 0.38
= 0.59
**Method 3** P(W or R) ~ P(at least 1 of W or R)
= 1 – P(W' and R')...**using the At–Least–One Principle!**
= 1 – 0.41
= 0.59
**8.** No, since 38% of voters are R *and* W.
**9.** P(R, given W) = P(R and W) / P(W)
= 0.38/0.52
**Explanation:** This situation is the OPPOSITE of that in b) above! Here, we are *given* that the voter was a woman. So we need to constrain our subset to 0.52 voters only! Out of those, 0.38 were Republican...
**10.** P = P(R and W') + P(R' and W)
= 0.07 + 0.14
= 0.21
**11.** If R and W were independent, then P(R | W) = P(R)
While Left side = P(R and W) / P(W) = 0.38/0.52,
Right Side = 0.45...which are NOT equal!
There, the 2 factors are NOT independent. **<––––––––––––––– CONCLUSION to answer the Q!**

**Problem.** Of all adults, the probability that someone is eligible for jury duty is 0.76, the probability that someone is married is 0.52, and 2/3rds of those *not* eligible for jury–duty are married.
**a)** Make a Probability Table...and then calculate the probability that an individual is not married.
**b)** Calculate the probability that an individual is married or eligible for jury duty.
**c)** Calculate the probability that an individual who is eligible for jury duty is married.
**d)** Calculate the probability that an individual is neither married nor eligible for jury duty.
**e)** Calculate the probability that an individual is not married, given that he/she was not eligible for jury duty.
**f)** Suppose someone is known to be married. Determine the probability that he/she is eligible for jury duty?
**g)** What is the probability that someone not eligible for jury duty is married?
**h) Consider the 2 events, E1: *Married* and *E2: Eligible for jury duty.*** Are E1 and E2 independent? Explain.

## Solution.

The # in parenthesis indicate the *order* in which the table was filled!

| | **M** | **M'** | **Total** |
|---|---|---|---|
| **JD** | 0.52 – 0.16 = 0.36 (6) | 0.76 – 0.36 = 0.40 | 0.76 (1) |
| **JD'** | 2/3 of 0.24 = 0.16 (5) | 0.24 – 0.16 = 0.08 | 0.24 (2) |
| **Total** | 0.52 (3) | 0.48 (4) | 1 |

**a)** Calculate the probability that an individual is not married.
**Interpretation:** P(M')
= P(JD and M') + P(JD' and M') ...**SHOW THIS STEP**: **examine** the table for this **obvious** step!
= 0.40 + 0.08
= 0.48
**b)** Calculate the probability that an individual is married *or* eligible for jury duty.
**Interpretation:** P(M or JD)
You *need* to master the following 3 ways!
**Method 1:** The General OR Rule [works **only** for 2 X 2 tables! Understand this well. The Rule **needs to be memorized!**]
P(M or JD)
= P(M) + P(JD) – P(M and JD)
= 0.52 + 0.76 – 0.36
= 0.92
**Method 2:** The *At–Least–One* Rule [works **only** for 2 X 2 tables! Understand this well.]
P(M or JD) ~ P(at least one of M or JD)
= 1 – P(neither of M or JD)
= 1 – P(M' and JD')
= 1 – 0.08
= 0.92

**Method 3:** Scan the table to examine which cells satisfy the condition of M or JD, *only one* of which needs to be True! [This common–sense approach works **always** even for tables with > 2 rows / columns! Understand this well.]
P(M or JD)
= P(M and JD') + P(M' and JD) + P(M and JD)
= 0.16 + 0.40 + 0.36
= 0.92
**caution!** Before you move on, insure that you've understood ALL 3 ways for the 2X2 OR probability.
**c)** Calculate the probability that an individual *who is eligible for jury duty* is married.
**Interpretation:** The Q asks us to *consider only those eligible for jury duty* [GIVEN], *and NOT the entire population of 100!*
**Note 1:** The GIVEN info *always* goes *after* the | and constitutes the denominator!
**Note 2:** Memorize the conditional probability formula,
P(A | B) = P(A and B)/P(B)
**Note 3:** Observe that P(A and B) is obtained by simply scanning the table for the *intersection* of A and B events.
**Note 4:** Do *not* use: P(A and B) = P(A)·P(B), which is true *only* for independent events! In 99.99% of the situations, the outcomes are *not* independent.
P(M | JD)
= P(M and JD) / P(JD)
= 0.36/0.76
**caution!** Before you move on, insure that you've understood how this conditional probability Q was phrased and interpreted. Read the Q again.
**d)** Calculate the probability that an individual is *neither* married *nor* eligible for jury duty.
**Interpretation:** P(M' and JD')
= 0.08
**e)** Calculate the probability that an individual is not married, *given that he/she was not eligible for jury duty*.
**Interpretation:** The Q asks us to *consider only those not eligible for jury duty* [GIVEN], *and NOT the entire population*.
P(M' | JD')
= P(M' and JD')/P(JD')
= 0.08/0.24
**caution!** Before you move on, insure that you've understood how this conditional probability Q was phrased and interpreted. Read the Q again.
**f)** Suppose someone is known to be married. Determine the probability that he/she is eligible for jury duty?
**Interpretation:** The Q asks us to *consider only those married* [GIVEN], *and NOT the entire population*.
P(JD | M)
= P(JD and M)/ P(M)
= 0.36/0.52
**caution!** Before you move on, insure that you've understood how this conditional probability Q was phrased and interpreted. Read the Q again.
**g)** What is the probability that someone not eligible for jury duty is married?
**Interpretation:** The Q asks us to *consider only those not eligible for jury duty* [GIVEN], *and NOT the entire population.*

P(M | JD')
= P(M and JD')/ P(JD')
= 0.16/0.24
**caution!** Before you move on, insure that you've understood how this conditional probability Q was phrased and interpreted. Read the Q again.
**h)** Are E1 and E2 independent? Explain.
*If* E1 and E2 were independent, then P(E1 | E2) = P(E1), by *definition* of independence.
[Alternately, P(E2 | E1) = P(E2)].
Left side: P(E1 and E2)/P(E2) = 0.36/0.76
Right side: P(E1) = 0.52
**Conclusion:** Since LHS ≠ RHS, E1 and E2 are not independent.

**Problem.** 28% of all women smoke, the probability that someone is over–weight is 0.35, and the probability that a woman who is overweight...is a smoker is 0.61. <––––––––––––––– **Careful!**
**a)** Construct a Probability Table.
**b)** If 5 individuals were chosen randomly, what is the probability that at least 1 shall be overweight?
**c)** What is the probability that a woman who is a smoker...is overweight?
**d)** What proportion of women belong from *at least one* of those categories: smoker, overweight?

## Solution.

**a)** Construct a Probability Table. The # in parenthesis indicate the *order* in which the table was filled!

| | S | S' | Total |
|---|---|---|---|
| **OW** | 0.61·0.35 = 0.2135 (5) | 0.35 – 0.2135 = 0.1365 (7 or 6) | 0.35 (3) |
| **OW'** | 0.28 – 0.2135 = 0.0665 (6 or 7) | 0.72 – 0.1365 *or* 0.65 – 0.0665 = 0.5835 (8) | 0.65 (4) |
| **Total** | 0.28 (1) | 0.72 (2) | 1 |

**caution!** Before you move on, insure that you've understood how this conditional probability problem was phrased and interpreted. Read the Q again.
**b)** If 5 individuals were chosen randomly, what is the probability that at least 1 shall be overweight?
**Interpretation:** P(at least 1 OW) Don't forget concepts from the previous topic!
= 1 – P(none are OW) **<––––––––––– SHOW THIS STEP!**
= 1 – P(OW')^5, for indep. events
First, P(OW')
= P(OW' and S) + P(OW' and S')...**SHOW THIS STEP**: **examine** the table for this **obvious** step!
= 0.0665 + 0.5835
= 0.6500
Substituting, P(at least 1 OW) = 1 – 0.65^5
**caution!** Before you move on, insure that you've understood how this problem was phrased and interpreted and resolved. Read it again.

**c)** P(OW | S) = P(OW and S) / P(S) = 0.2135 / 0.28

**d) Method I** P(OW or S) = P(OW) + P(S) – P(OW and S) = 0.35 + 0.28 – 0.2135
**Method II** P(OW or S) ~ P(at least 1 of OW and S) = 1 – P(belongs to neither category) = 1 – P(OW' and S') = 1 – 0.5835
**Method III** P(OW or S) = P(OW and S) + P(OW and S') + P(OW' and S') = 0.2135 + 0.1365 + 0.0665

**Translate the following into probability statements.** For each Q, ask yourselves: **What are the sub–sets involved? The GIVEN information ~ Subset goes AFTER the |.**
**Problem.** The probability that a frequent traveler to Africa carries the West Nile virus is 1.23% while amongst the less frequent travelers it's 0.054%. **Tip!** What are the sub–sets involved?
**Solution.** P(WNV | FT) = 1.23%; P(WNV | LFT) = 0.054%
**Problem.** 0.4% of those that are hygienic while cooking contracted food–poisoning while 23% of those that weren't hygienic contracted food–poisoning. **Tip!** What are the sub–sets involved?
**Solution.** P(FP | H) = 0.4%; P(FP | H') = 23%

**Problem. Translate the following into probability statements.** For each Q, ask yourselves: **What are the sub–sets involved? The GIVEN information ~ Subset goes AFTER the |.**
1. 1.5% of LSHS students get into UCLA last year while 0.02% got into Berkeley.
2. Of the late risers, 10% were early to school; 5% of the early risers were late to school.
3. 97% of drivers that *were* sober were found to be sober; 9% of drunk drivers were found to sober.
4. 2% of cancer–free individuals were diagnosed as suffering from it; of those with cancer, 85% were deemed as suffering from it.
5. 10% of packages are delivered by company A, 20% by B, 40% by C and the rest by D. From experience, 1%, 2%, 3% and 4% of packages are delivered by A, B, C and D, respectively, are delivered late.
6. 20% of children are obese whereas 34% of adults were.
7. Amongst the Independent voters, the proportion of Blacks was 16%; also, 23% of Independents were Hispanic
8. 76% of customers who were unhappy were Asian; 48% of happy customers were Asian
9. Of the Females, 18% had lung cancer; amongst the males, 11% had lung cancer

## Solution.

**Note. Your symbols <u>may</u> not match mine *perfectly*…and that's OK. But they ought to be in the right spots!**
1. P(UCLA | LSHS) = 1.5%, P( B | LSHS) = 0.02%
2. P(ES | LR) = 10%; P(LS | ER) = 5%
3. **A ~ Actually F ~ Found** P(FS | AS) = 97%, P(FS | AD) = 9%
4. P(+ | C') = 2%; P(+ | C) = 85%
5. P(A) = 10%; P(B) = 20%; P(C) = 40%; P(D) = 30%; P(L | A) = 1%; P(L | B) = 2%; P(L | C) = 3%; P(L | D) = 4%
6. P(O | C) = 20%; P(O | A) = 34%
7. P(B | I) = 16%; P(H | I) = 23%
8. P(A | H') = 76%; P(A | H) = 48%
9. P(LC | F) = 18%; P(LC | M) = 11%

**Problem. Fill in the blanks suitably:**

1. P(A | B) = ____
2. P(A and B) = P(A) · P(B) **only if** A and B are ____
3. ____ = P(M | N)
4. P(L and R) / P(R) = ____
5. P(K | S) · ____ = P(K and S)
6. **Sometimes True or Always True?** P(A or B) = P(A) + P(B) **If Sometimes,** specify when ____.
7. P(T and 0) = P(T) · ____
8. P(L and M) = P(L | M) · ____
9. P(S | F) = ____ / P(F)
10. P(D and E) = ____ · P(D | E)
11. ____ · P(G) = P(G and X)
12. Write the definition of Mutually Exclusive events.
13. **Sometimes True or Always True?** P(A | B) = P(A and B) / P(B) **If Sometimes,** specify when ____.
14. For Independent Events, P(M | N) = ?
15. For Mutually Exclusive events A and B: P(A and B) = ? [Sketch a Venn Diagram to help you!]
16. For Mutually Exclusive events A and B: P(A or B) = ? [Sketch a Venn Diagram to help you!]
17. Write the definition of Independent events.
18. **Sometimes True or Always True?** P(A and B) = P(A)·P(B) **If Sometimes,** specify when ____.
19. For Independent Events A and B, P(A and B) = ?
20. If 2 events cannot occur at the same time theyre called ?
21. If events are such that one's outcome does not affect / influence the other, theyre called ?
22. P(A and B) = 0 **only if** A and B are ?
23. **Sometimes True or Always True?** P(A and B) = P(B) · P(A | B). **If Sometimes,** specify when ____.
24. Write another term for Mutually Exclusive events.
25. What is the General OR Rule for ***non***–Mutually Exclusive Events: P(A or B) = ? [Sketch a Venn Diagram to help you: **draw it so that there IS an overlap**!]
26. **Sometimes True or Always True?** P(A | B) = P(A) **If Sometimes,** specify when ____.
27. If A and B are independent events, what are 2 ways to check for it [using it's definition]?

**Solution.**

1. P(A | B) = **P(A and B) / P(B)**
2. P(A and B) = P(A) · P(B) **only if** A and B are **independent.**
3. **P(M and N) / P(N)** = P(M | N)
4. P(L and R) / P(R) = **P(L | R)**
5. P(K | S) · **P(S)** = P(K and S)
6. **Sometimes True or Always True?** P(A or B) = P(A) + P(B) **Sometimes,** specify when **when A and B are mutually exclusive.**
7. P(T and 0) = P(T) · **P(T | O)**
8. P(L and M) = P(L | M) · **P(M)**
9. P(S | F) = **P(S and F)** / P(F)
10. P(D and E) = **P(E)** · P(D | E)
11. **P(X | G)** · P(G) = P(G and X)
12. Write the definition of Mutually Exclusive events. **Events that cannot occur at the same time.**

13. **Sometimes True or Always True?** P(A | B) = P(A and B) / P(B) **Always true.** This is a general formula with no conditions or specifications!
14. For Independent Events, P(M | N) = **P(M)**
15. For Mutually Exclusive events A and B: P(A and B) = **Zero.**
16. For Mutually Exclusive events A and B:P(A or B) = **P(A) + P(B) because of #14 and the OR Rule!**
17. Write the definition of Independent events. **Events whose outcomes don't affect each other.**
18**. Sometimes True or Always True?** P(A and B) = P(A) · P(B) **Sometimes when A and B are Independent.**
19. For Independent Events A and B, P(A and B) = **P(A) · P(B)**
20. If 2 events cannot occur at the same time theyre called **Disjoint / Mutually Exclusive Events.**
21. If events are such that one's outcome does not affect / influence the other, theyre called **Independent Events.**
22. P(A and B) = 0 **only if** A and B are **Disjoint or Mutually Exclusive.**
23. **Sometimes True or Always True?** P(A and B) = P(B) · P(A | B). **Always [because this is just the cross-mutiplied version of P(A | B), which is a general formula of conditional probability!]**
24. Write another term for Mutually Exclusive events. **Disjoint.**
25. What is the General OR Rule for ***non***-Mutually Exclusive Events: P(A or B) = **|P(A) + P(B) - P(A and B)**
26. **Sometimes True or Always True?** P(A | B) = P(A) **Sometimes, for Independent Events.**
27. If A and B are independent events, what are 2 ways to check for it [using it's definition]? **P(A | B) = P(A)** or **P(B | A) = P(B)** or **P(A and B) = P(A) · P(B)**

**Problem.** 1. In a certain town, there are only 2 major crimes: burglaries and assault: 43% are burglaries and the rest are assaults. Also, 21% of burglaries are committed by children, 45% of burglaries are committed by teenagers and the **rest** [~ left–overs!] by adults. 6% of assaults are committed by children, 15% of assaults are committed by teenagers and the rest by adults.
a) Carefully, express each of the probabilities above in Notation form. **Tip!** 2 are simple probabilities **[e.g. P(A)],** 6 of them are conditional **[e.g. P(L | M)]**!
b) Examining
a) and using the Conditional Probability Multiplication Rule [we learnt today!] calculate 6 AND probabilities.
c) Create a suitable 3 X 2 table and fill in all the blanks using a) and b). **Tip**! The AND probabilities go INSIDE while the simple probabilities go outside.
d) What is the probability that a child is involved in a crime? **Show work.**
e) Suppose a major crime is committed and the culprit is found to be a teenager. What is the probability that he / she committed an assault?
f) Suppose 5 major crime records are randomly chosen. What is the probability that at least one is an assault?

## Solution.

a) P(B) = 0.43, P(A) = 0.57
P(C | B) = 0.21, P(T | B) = 0.45, P(Ad | B) = 1 – 0.21 – 0.45 = 0.34
P(C | A) = 0.06, P(T | A) = 0.15, P(Ad | A) = 1 – 0.06 – 0.15 = 0.79
b) P(C and B) = 0.0903, P(T and B) = 0.1935, P(Ad and B) = 0.1462
P(C and A) = 0.0342, P(T and A) = 0.0855, P(Ad and B) = 0.4503

c)

| | C | T | Ad | |
|---|---|---|---|---|
| B | 0.0903 | 0.1935 | 0.1462 | 0.43 |
| A | 0.0342 | 0.0855 | 0.4503 | 0.57 |
| | 0.1245 | 0.279 | 0.5965 | 1 |

d) P(C) = P(C and B) + P(C and A) = 0.0903 + 0.0342 = 0.1245
e) P(A | T) = P(A and T) / P(T) = 0.0855/0.279
f) P(at least 1 A) = 1 – $P(A')^5$, assuming independence = 1 – $0.43^5$ = 0.9852

**Problem.** An unmanned monitoring system uses high-tech video equipment and microprocessors to detect intruders. A prototype system has been developed and is in use outdoors at a weapons munitions plant. The system is determined to detect intruders with a probability of 90%. However, the design engineers expect this probability to vary with weather conditions. The system automatically records the weather condition each time an intruder is detected. Based on a series of controlled tests, in which an intruder was released at the plant under various weather conditions, the following information is available:

- When the intruder was, in fact, detected by the system, the weather was clear 75% of the time, cloudy 20% of the time, and raining 5% of the time.
- When the system failed to detect the intruder, 60% of the days were clear, 30% cloudy and 10% rainy.

Calculate the probability of detecting an intruder, under rainy weather conditions.
**Tip!** Write the 7 probabilities using Notation, then make a table. Then, interpret the Q using notation and solve!

**Solution.**
P(D) = 0.9, P(Clr | D) = 0.75, P(Cld | D) = 0.20, P(R | D) = 0.05
P(Clr | D') = 0.6, P(Cld | D) = 0.30, P(R | D) = 0.10
Use the multiplication rule to find: P(D and Clr), P(D and Cld), P(D and R), P(D' and Clr), P(D' and Cld), P(D' and R):
The # in parenthesis indicate the *order* in which the table was filled!

| | Clear | Cloudy | Raining | Total |
|---|---|---|---|---|
| D | 0.675 (3)<br>= 0.75·0.9 | 0.18 (4)<br>= 0.2·0.90 | 0.045 (5)<br>= 0.05·0.90 | 0.90 (1) |
| D' | 0.60·0.1 = 0.06 (6) | 0.3·0.1 = 0.03 (7) | 0.10·0.1 = 0.01 (8) | 0.10 (2) |
| | 0.735 (9) | 0.21 (10) | 0.055 (11) | 1 |

P(D | Rain) = P(D and Rain) / P(Rain) = 0.045/0.055

**Problem.** People with type O-negative blood are universal donors. That is, any patient can receive a transfusion of O-negative blood. Only 7.2% of the American population have O-negative blood. If 4 people appear at random to give blood, what is the probability that

**a)** at least 1 of them is a universal blood donor?

**b)** only 1 of them is a universal blood donor?

**c)** none of them is a universal blood donor?

**d)** at most one of them is a universal blood donor? [**Careful!** Which 2 scenarios apply...that we already have?!]

P(O) = 0.072 and P(O') = 0.928

**Solution.**

**a)** P(at least 1 O) = 1 – P(none are O)

= 1 – P(O', O', O', O')

= 1 – $P(O')^4$, assuming Independence

= 1 – $(0.928)^4$ = 52.63%

**b)** P(exactly 1 0) = P(O, O', O', O') + P(O', O, O', O') + P(O', O', O, O') + P(O', O', O', O)

= 4· P(O, O', O', O') since the 4 probabilities are identical!

= $4 \cdot P(O) \cdot P(O')^3$, assuming Independence

= $4 \cdot 0.072 \cdot 0.928^3$

**c)** P(none are O) = P(O', O', O', O')

= $P(O')^4$, assuming Independence

= $0.928^4$

**d)** P(at most one O) = P(none are O) + P(exactly 1 O)

= $0.928^4 + 4 \cdot 0.072 \cdot 0.928^3$

**Problem.** 84% of US adults watch movies on DVD once a week; 73% of those that watch movies on DVD once a week, order pizzas once a week. Also, 10% of US adults order neither.

a) Write the probabilities using suitable notation. Perform certain calculations and make a table.

b) What proportion of those that buy pizza once a week...watch movies on DVD once a week? Translate the Q into Notation carefully, $1^{st}$.

c) What is the probability that a randomly selected US adult watches movies on DVD once a week or orders pizza once a week *but not both*? Translate the Q into Notation carefully, $1^{st}$.

**Solution.**

a) P(DVD) = 0.84, P(Pz | DVD) = 0.73, P(DVD' and Pz') = 0.10.

Therefore, P(DVD and Pz) = 0.6132

Make a table!

b) P(DVD | Pz) = **Finish.**

c) P(DVD and Pz') + P(DVD' and Pz) = **Finish.**

**Problem.** Here is the distribution of the adjusted gross income (in thousands of dollars) reported on individual federal income tax returns in a recent year:

| income | < 25 | 25– 49 | 50– 99 | 100– 499 | ≥ 500 |
|---|---|---|---|---|---|
| probability | 0.431 | 0.248 | 0.215 | 0.100 | 0.006 |

(a) What is the probability that a randomly chosen return shows an adjusted gross income of \$50,000 or more?
(b) Given that a return shows an income of at least \$50,000, what is the conditional probability that the income is at least \$100,000?

## Solution.

a) P(AGI ≥ \$50K)
= P(AGI = \$50–99K)+ P(AGI = \$100–499K) + P(AGI ≥ \$500K)
= 0.215 + 0.1 + 0.006
b) P(AGI ≥ \$100K | AGI ≥ 50K)
= P(AGI ≥ \$100K **and** AGI ≥ 50K) / P(AGI ≥ 50K)
= P(AGI ≥ \$100K) / P(AGI ≥ 50K)
**Note:** on a number line, numbers satisfying BOTH:
AGI ≥ \$100K **and** AGI ≥ 50K
are simply those satisfying: AGI ≥ \$100K.
Do **<u>not</u>** *mechanically* multiply the probabilities: P(AGI ≥ \$100K) · P(AGI ≥ 50K)
These are NOT Independent Events!

——————————————

**Problem.** Illegal music downloading has become a big problem: 29% of Internet users download music files, and 67% of downloaders say they don't care if the music is copyrighted. What percent of Internet users download music *and* don't care if it's copyrighted?
**Tip!** Write the information given in terms of probabilities, then interpret the Q suitably and solve.

## Solution.

P(DM) = 0.29; P(DC | DM) = 0.67.
Therefore P(DM and DC) = P(DM)·P(DC | DM) = 0.29·0.67

**Tip!** *Alternately,* one can construct an incomplete TABLE.

| | **Don't Care** | **Care** | **Total** |
|---|---|---|---|
| **D** | 67% of 0.29 | | 0.29 |
| **D'** | | | 0.71 |
| **Total** | | | 1 |

Required: P(D and Don't Care) = 67% of 0.29

**Problem.** In a recent month, 88% of automobile drivers filled their vehicles with regular gasoline, 2% purchased midgrade gas, and 10% bought premium gas. Of those who bought regular gas, 28% paid with a credit card; of customers who bought midgrade and premium gas, 34% and 42%, respectively, paid with a credit card. Suppose we select a customer at random.
**a)** What's the probability that the customer paid with a credit card?
**b)** Of the customers that paid with a credit card, what proportion bought premium gas?

## Solution.

a) P(CC) = P(CC and R) + P(CC and MG) + P(CC and P) = 0.2464 + 0.0068 + 0.042 = 0.295
b) P(P | CC) = P(P and CC)/P(CC) = 0.042/0.295 = 0.1423
**The (#) indicate the ORDER in which the table was filled...**

| | CC | CC' | |
|---|---|---|---|
| RG | 28% of 0.88 (4) | (7) | 0.88 (1) |
| MG | 34% of 0.02 (5) | (8) | 0.02 (2) |
| PG | 42% of 0.1 (6) | (9) | 0.1 (3) |
| | (10) | (11) | 1 |

**a)** P(CC)
= P(CC and RG) + P(CC and MG) + P(CC and PG)
**b)** P(PG | CC)

## Extra Practice:

1. Meteorologists state that there's a 20% that it will rain on a certain day of the week. From experience, a teacher knows that, in general, there's a 90% probability that a student shall be present in class. If the probability that it rains and the student is present is 30%, find the probability that it rains *or* the student is present.

2. You have 10 batteries, of which 6 are Good. You pick 2 batteries, one after the other, without replacement.
a) What is the probability that *both* are Good?
b) What is the probability that you select *at least* 1 Good battery?

3. Jennifer is deciding what to cook for dinner. The probabilities that she prepares a certain dish are given: P(Rice) = 0.8; P(Soup) = 0.7; P(Pasta) = 0.95; P(Meat) = 0.65. Her choices are independent of each other.
a) What is the probability that Jennifer shall prepare the Soup and the Pasta but not the Rice and Meat?
b) What is the probability that Jennifer shall prepare *at least* 1 of those dishes?

4. A grocery store has 2 kinds of apples, Fuji and Gala, mixed in a large basket.
There are 10 Large Fuji and 15 Small Fuji apples. Also, there are 8 Large Gala and 12 Small Gala apples.
a) If you select 1 apple randomly, what is the probability that it is Large?
b) If you select 1 apple randomly, what is the probability that it is a Gala?
c) If you select 1 apple randomly and find it it to be small, what is the probability that it is a Fuji?
d) If you select 1 apple randomly, what is the probability that they are Fuji *or* Large?
e) If you select 2 apples randomly, without replacement, what is the probability that both shall be Large?

5. Suppose 23% of individuals, in general, jog. Also, 57% of joggers and 13% of non–joggers develop knee disorders by age 65. The following table depicts the probabilities of Jogging and developing Knee Disorders by 65.

| | J | J' | Total |
|---|---|---|---|
| KD | 0.1311 | 0.1001 | 0.2312 |
| KD' | 0.0989 | 0.6699 | 0.7688 |
| Total | 0.2300 | 0.7700 | 1.0000 |

a) If an individual is randomly selected, what is the probability that he jogs and shall develop a knee disorder by 65 years?
b) If an individual is randomly selected, what is the probability that he jogs *or* shall develop a knee disorder by 65 years?
c) If a jogger is randomly selected, what is the probability that he shall develop a knee disorder by 65 years?
d) If 5 individuals are randomly selected, what is the probability that *all* of them shall develop a knee disorder by 65 years?

8. The probability for snow today is 40%. The probability for snow tomorrow is 35%. The probability for snow today or tomorrow is 17%. Find the probability that
a) it will snow today or tomorrow.
b) it will snow exactly one day.
c) it will snow only today.

9. The probability that it snows any day this week is 40%. Assume a 5–day week and that the snow days occur independently of each other.
(i) Find the probability that it will snow on Tuesday.
(ii) Find the probability that it will snow on Monday or Tuesday.
(iii) Find the probability that it will snow on Monday or Tuesday but not on both days.
(iv) Find the probability that it will *not* snow on Thursday only.
(v) Find the probability that it will snow on neither Wednesday nor Thursday.
(vi) Find the probability that it will *not* snow on Thursday <u>only</u>.
(vii) Find the probability that it will *not* snow on Monday and Thursday <u>only</u>.
(viii) Find the probability that the 1$^{st}$ day it snows is on Thursday.
(ix) Find the probability that it shall snow on exactly 2 days of the week.
(x) Find the probability that it will snow on at least 1 day this week.

10. A fellow has 4 cars. The probability that they break–down is as:
Ford, 8%; GM, 4%; Toyota, 5%; Nissan: 3%
i) What is the probability that the Toyota does not break down?
ii) What is the probability that *only* the Toyota doesn't break down?
iii) What is the probability that *only* the Ford breaks down?
iv) What is the probability that neither the Toyota nor the Ford breaks down?
v) What is the probability that the Toyota or the Ford breaks down?

vi) What is the probability that *none* of the cars break down?
vii) What is the probability that at least 1 car breaks down?

11. There are 8 large red mangoes and 7 small red mangoes in one basket; there are 6 large yellow mangoes and 11 small yellow mangoes in another basket.
a) If 1 mango is randomly chosen, what is the probability that it is large?
b) If 1 mango is randomly chosen, what is the probability that it is small and red?
c) If a large mango is randomly chosen, what is the probability that it is yellow?
d) If a small mango is randomly chosen, what is the probability that it is red?
e) If a mango is randomly chosen, what is the probability that it is not large?

Two mangoes are chosen, one from each basket. What is the probability that
f) both are small?
g) both are yellow?
h) at least 1 is red?
i) at least 1 is large?
j) only the $1^{st}$ is red?
k) only the $2^{nd}$ is large?
l) only 1 of them is red? [Careful!]

12. In an election, 52% of voters were women; 45% of voters voted Republican; and 38% were women and Republican.
a) Construct and fill a table first to illustrate the situation.
b) Determine the probability that a voter was a woman but not a Republican.
c) Determine the probability that a voter was a woman if it is known that she was a Republican.
d) Determine the probability that a voter was not a woman, given that the voter was not Republican.
e) Determine the probability that a voter was neither a woman nor a Republican.
f) Determine the probability that a voter was male and a Republican.
g) Calculate the probability that a voter was a woman or a Republican.
h) What is the probability that a voter who is a woman would be a Republican?
i) What is the probability that a Republican…is a woman?
j) What is the probability that a randomly chosen individual would be a Republican or a woman but not both?
k) What is the probability that a woman…is not a Republican?
l) If 2 individuals are randomly selected, what is the probability that both are women?
m) If 3 individuals are randomly selected, what is the probability that at least 1 is Republican?
n) If 2 individuals are randomly selected, what is the probability that neither is Republican?

13. Annie is on the swim team. There are 25 girls on the team. Five will be picked at random to attend a swim seminar. What is the probability that Annie *will not be* picked?

14. Of all adults, the probability that someone is eligible for jury duty is 0.76, the probability that someone is married is 0.52, and the probability that you're married and eligible for jury duty is 0.40. Construct and fill a table first to illustrate the situation. Calculate the probability that an individual
a) is not married.
b) is married or eligible for jury duty.

c) who is eligible for jury duty is married.
d) is neither married nor eligible for jury duty.
e) is not married, given that he/she was not eligible for jury duty.
f) Suppose someone is known to be married. Determine the probability that he/she is eligible for jury duty?
g) What is the probability that someone not eligible for jury duty is married?
h) If 2 individuals are randomly selected, what is the probability that both shall be married?
m) If 4 individuals are randomly selected, what is the probability that at least 1 is eligible for jury duty?
n) If 2 individuals are randomly selected, what is the probability that exactly 1 is married?

15. I own a washing machine, a blender, and a dishwasher. The probability that they are working are, respectively: 70%, 95%, 98%. What is the probability that
a) none of them are working?
b) at least 1 of them is working?
c) at least 1 of them is not working?
d) only the blender works?

16. If 2 dice are thrown, what is the probability that
a) the 1$^{st}$ die is an odd number
b) only the 1$^{st}$ die is an odd number
c) both dice are 5 or higher
d) both dice show 3 or lower
e) their sum exceeds 8
f) their sum exceeds 8, given that the 1$^{st}$ die is a 6
g) their sum exceeds 8, given that the 1$^{st}$ die is an even number
h) their sum is 4 or less, given that the 2$^{nd}$ die is an odd number
i) the 1$^{st}$ is even, given that the 2$^{nd}$ is a 5.

17. The 2–way table gives suicides committed in a recent year classified by Gender and Firearm used.

| | Male | Female | Total |
|---|---|---|---|
| Firearm | 16,381 | 2,559 | 18,940 |
| Other | 9, 034 | 3,536 | 12, 570 |
| Total | 25, 415 | 6, 095 | 31, 510 |

If a suicide was randomly chosen,
a) find the probability that a firearm was used.
b) find the probability that a female…used a firearm.
c) given that the suicide committed by firearm, what was the probability that the victim was male.
If 3 suicides were randomly chosen, find the probability that
d) only the 1$^{st}$ was male.
e) at least 1 was committed by a Firearm.

18. The probability that a fellow is a Democrat is 45%, that he votes is 65%, and that he is a democrat OR votes is 90%. Find the probability that a fellow is a Democrat, given that he votes.

19. The probability that Melisa loses her purse at the Mall is 0.3, and the probability that she loses her cell–phone at the Mall is 0.6. Assuming that the 2 events are independent, find the probability that

(a) Melisa loses her purse at the Mall but does <u>not</u> lose her cell–phone.
(b) Melisa loses <u>both</u>, her purse and her cell–phone, at the Mall
(c) Melisa does not lose either item at the Mall
(d) Melisa loses her purse, if it is known that she lost her cell–phone

20. In an assembly plant, the probability that Part 1 arrives on time is 0.9, and the probability that Part 2 arrives on time, given that Part 1 did, is 0.8. Find the probability that Part 1 arrives on time, given that Part 2 did.

21. The probability that Tom takes the bus to school 0.7, and given that Tom took the bus to school, the probability that he gets picked up by car for his return is 0.6. Find the probability that Tom takes the bus to school and he gets picked up by car for his return.

22. There are 6 Red and 7 Yellow mangoes in a crate, all mixed up. You prefer Red mangoes. You select 3 mangoes, one after the other. What is the probability that

a) none of the ones you select are the kind you want?
b) at least 1 mango is the type you prefer?
c) it takes you the 3rd attempt to get the one you like?
d) you get <u>exactly one</u> Red mango?

23. A 10 sided die numbered from 1–10 is rolled. Find the probability that you get

a) an even number
b) a number less than 7
c) an odd number or a multiple of 3
d) an even number or a number less than 4