Praxis Mathematics 5161 Independent Practice

Have a stab at these Qs for additional practice!

1. A sector shaped like a piece of a pie is cut from a circle of radius r. The outer circular arc of the sector has length s. If the total perimeter of the sector (i.e. $2r + s$) equals $100\ cm$, what values of r and s shall maximize the area of the sector?

2. If 1, 1/2 and -3 are x-intercepts of a polynomial function $f(x) = ax^3 + bx^2 + cx + 3$, find $f(x)$ and graph it.

3. Find:
   a. the equation of the parabola passing through $(0,5), (2,5)$ and $(-2,21)$.
   b. the equation of the line with x-intercept 5/8 making a triangular area of 25/16 in the 4th Quadrant.
   c. at what point(s) does the line intersect the parabola?

4. A rectangle is to be $inscribed$ in a semicircle of radius $r\ cm$. If the height of the rectangle is h, write an expression in terms of r and h for the Area and Perimeter of the rectangle. What dimensions of the rectangle yield the maximum Area?

5. The fat and corresponding calorie content for $5oz$ of 7 kinds of pizza are given below:

Fat (g)	9	16	14	9	10	11	13
Calories	306	383	339	328	310	314	350

   a. Plot the points using an appropriate scale.
   b. Calculate the Least Squares Regression Line (LSRL) by hand.
   c. Interpret the slope and y-intercept of the LSRL.

6. Solve the trigonometric equation: $2\cos^2 x \sin x + 3 \cos x \sin x = -\sin x$ in the interval $[-2\pi, 2\pi]$.

7. On the same graph, plot the following functions:
   a. $f(x) = x^2 - 2x - 8$

b. $g(x) = \sqrt{f(x)}$
c. $h(x) = 1/f(x)$

8. ΔCAB is right-angled at A with $AC = 8$ and $AB = 12$. Points F, E and D lie on sides AB, BC and CA respectively such that $AFED$ is a rectangle. If $AF \approx x$, find an expression for the Area of rectangle $AFED$ in terms of x.

9. The number of bacteria, in thousands, in a Petri dish is given by the function: $N(T) = 15T^2 - 100T + 1000, 5 \leq T \leq 20$ where T denotes the temperature in Celsius. The temperature of the Petri dish is a function of time, t, in hours: $T(t) = 3t + 7,\ 0 < t < 5$
   a. Find the number of bacteria in the Petri dish after 3 hours have elapsed.
   b. If 1,500,000 bacteria are found in the Petri dish, how many hours must have elapsed?

10. A quality control inspector examines DVD players for a complete inspection as they come off an assembly line. It is known that 10% of all items are defective. Also, 60% of all defective items and 20% of all good items undergo a complete inspection.
   a. If a randomly selected DVD player undergoes a complete inspection, what is the probability that it is defective?
   b. If a certain DVD player did not undergo a complete inspection, what is the probability that it is not defective?

11. Prove the Law of Cosines: If a, b and c are sides of triangle ABC and x is the angle opposite c, then $c^2 = a^2 + b^2 - 2ab\cos x$

12. Find the number of integers between 50 and 500 $[inclusive]$ divisible by 5 or 7.

13. Find the area of the largest circle that would fit $inside$ an equilateral triangle with area $36\ cm$.

14. A square is inscribed inside an equilateral triangle such that 2 of its vertices lie on 1 side of the triangle, and the other 2 vertices lie ON the other 2 sides of the triangle. Find the $ratio$ of the $sum\ of\ the\ areas\ of\ the\ 2\ triangles$ (on the bottom, to the right & left of the square) to $the\ area\ of\ the\ triangle\ on\ top$ (the equilateral triangle on top of the square). Using the figure above, if the area of the square is 3, what is the perimeter of the triangle?

15. The distribution of colors for plain M&Ms is given below:
Red (20%), Yellow (20%), Green (10%), Orange(10%), Brown (30%) and Blue (10%). If a random sample of 75 peanut M&Ms yields the following distribution of colors: Red (14), Yellow (15), Green (6), Orange(8), Brown (23) and Blue (9), is the distribution of colors of peanut M&Ms the same as that of plain M&Ms?
    a. State an appropriate hypothesis.
    b. Perform a Chi-Square test to test the hypothesis at the 5% significance level.
    c. Interpret the *p-value* in the context of the question.

16. Show that the Area of the region bounded by the graph $f(x) = 3x - x^2$ and the x-axis between $x = 1$ and $x = 2$ is 13/6.

17. Find the magnitude and direction of the vector $v = -i + j$.

18. A computer manufacturer determines that it can sell 4000 machines at $750, and that for each $25 that the price is increased 20 fewer computers are sold. Let n represent the number of $25 increases in the price. Express the total revenue of computer sales as a function of n and find how much the manufacturer should charge to maximize his revenue.

19. Three *congruent* circles are arranged to touch each other tangentially (i.e. each touches the other two at one point only). A rectangle is drawn to circumscribe the 3 circles (i.e. its sides act as tangents to the circles). If the radius of each circle is $1\ cm$, find the area and perimeter of the rectangle.

20. The costs per load (in cents) of 35 detergents tested by Consumer Reports are shown below:

Cost	13-19.9	20-26.9	27-33.9	34-40.9	41-47.9	48-54.9	55-61.9	62-68.9
Frequency	2	7	12	5	6	1	0	2

a) Draw a frequency and cumulative frequency distribution for the data.
b) Calculate the mean, median and modal costs per load of the 35 detergents. Interpret each.
c) Compute the standard deviation and variance. Interpret each.

21. Graph the rational function: $f(x) = \frac{(x^2 + 2x - 15)}{x^3 - 7x^2 + 12z}$ stating its

a. Domain
b. x- and y-intercepts
c. Asymptotes

22. Find the Greatest Common Factor and Least Common Multiple of 5160, 5640, 4920, 4680 and 3720. [Express the LCM as a product of primes.]

23. What does the following construction PO produce? Justify.

Step 1. Draw a horizontal line, l, and a point, P, above it.
Step 2. With P as center, draw an arc (Arc 1) cutting line, l, at points A and B.
Step 3. With A as center and radius > AB, draw a wide arc (Arc 2) on the other side of line, l, across P.
Step 4. With B as center and keeping the radius the same, draw an arc (Arc 3) cutting Arc 2 at O.
Step 5. Join PO.

24. The height of a rock thrown vertically upward on the moon with a velocity of 24 m/sec is given by $h(t) = 24t - 0.8t^2$, t being the time elapsed in seconds.
a. What is the rock's velocity and acceleration after t seconds?
b. How long does it take to reach the highest point? How high does the rock go?
c. How long does the rock take to reach half its maximum height? What is its velocity and acceleration at that moment?
d. How long is the rock aloft for its entire journey?
e. With what velocity does it strike the surface of the moon? What is its acceleration then?

25. A parabola its x-intercepts at –1 and 3. If the minimum value of the function is –2, find its equation.

26. Determine the constant term of the polynomial with integer coefficients of lowest degree with roots ± 1 and $(-3 + \sqrt{7})/2$, and whose leading coefficient is 3.

27. Imagine an Isosceles Triangle inscribed in a Semi-circle, with the longest side obviously the diameter. Now, using each congruent side of the triangle as base (i.e. diameter) 2 smaller semicircles are drawn. Find the $ratio$ of the Area of the

Triangle to the Sum of the Areas of the 2 semi-circles that lie $outside$ the large semi-circle.

28. A company borrows \$500,000 to expand its product line. Some of the money is borrowed at 7%, some at 8% and some at 10%. If the annual interest is \$63,000 and the amount borrowed at 7% is 4 times that borrowed at 10%, express the system of equations to be solved in Matrix Form.

29. Find the area of a circle with perimeter same as that of a square with diagonal 10 cm.

30. $ABCD$ is a rectangle with AB & CD as its long sides. It is enclosed in a circle with center, O. The measure of angle AOB is 120 degrees and the length of AB is 12 cm. Find the Area of the region $inside\ the\ circle$ but $outside\ the\ rectangle$.

31. A square sheet of paper of side, A, is snipped at a distance of C units from the corners to produce a regular octagon of side, B units. Find B and C in terms of A.

32. Suppose we're interested in finding out about the support a candidate John Smith has, and we randomly interview 12 voters. Assume his overall approval rating to be 41%.
    a. Describe how the 4 conditions of a Binomial situation are met in context.
    b. Define a suitable Binomial random variable, X.
    c. What is the probability exactly 6 chaps approve of John Smith?
    d. What is the probability more than 8 chaps approve of John Smith?
    e. What is the probability fewer than 3 chaps approve of John Smith?
    f. What is the probability no more than 5 chaps approve of John Smith?
    g. What is the probability of getting at least 1 John Smith supporter?
    h. In a sample of 12 individuals, how many would you expect to be John Smith supporters? What is the standard deviation of the number of supporters?
    i. Suppose voters are repeatedly asked for their preferences. What is the probability that the 1st John Smith voter shall be the 5th one chosen?
    j. Suppose voters are asked for their preferences, one after the other. What is the probability that the 4th John Smith voter shall be the 15th one chosen?

33. A logistic growth model of the form $y = \frac{A}{(B + Ce^{-Kx})}$ can be used to describe the proportion of households, y, that owns a DVD at year, x, where A, B, C and K are constants. Express x as a function of y.

34. For what value(s) of K will the parabola $y^2 - 6y + x + K = 0$ have exactly 1 y-intercept? Find the intercept.

35. For very large values of x, the graph of $f(x) = \frac{x^2}{(x+1)}$ behaves as that of a line $y = mx + b$. Find m and b.

36. A water bucket containing 10 gallons of water develops a leak. The volume, $V(t)$ of water in the bucket t seconds later is given by $V(t) = 10(1 - (t/100))^2$ until the bucket is empty 100 seconds later.
   a. At what rate is the water leaking after exactly 1 minute?
   b. When is the instantaneous rate of change of volume equal to the average rate of change of volume from $t = 0$ to $t = 10$ seconds?
   c. At the instant described in part b, how much water is in the tank?

37. Find the area of the largest equilateral triangle that would fit $inside$ a circle of circumference $36\pi\ cm$.

38. If point (3, 2) lies on the graph of the inverse of $f(x) = 2x^3 + x + A$, find A.

39. In triangle ABC, BO and CO are the internal bisectors of Angle B and Angle C. Prove that Angle BOC = 90 + ½ ∠A.

40. A 12-sided regular convex polygon of side 10 cm is circumscribed by a circle.
   a. Find the Area of the region inside the circle but outside the polygon.
   b. If another circle were inscribed into the above polygon, find the Area of the region outside the circle but lying inside the polygon.

41. Water flows into a tank $150m$ long and $100m$ wide through a pipe whose cross section is $0.2m$ by $0.15m$ at a speed of $15km$ per hour. In what time will the water be 3 meters deep?

42. The heights of male 18-24 year olds in the US is roughly normally distributed with a mean of $70.1in$ and a standard deviation of $2.7in$.
   a. If a US male in the 18-24 age group were selected at random, calculate the probability that he is less than $66in$ tall?

b. Out of the roughly 13 million US males between 18-24 years, roughly how many have heights between $68in$ and $71in$?
c. Calculate the height that constitutes the $70th$ percentile for this population.

43. What is the point of intersection of $f(x) = ½\,x + 3/2$ and its inverse?

44. What shape does the graph $|y| + |x| \leq 3$ resemble?

45. ABC is an isosceles triangle inscribed in a circle with center, O. (Assume A is the top vertex, B the vertex on the bottom right, C, on the bottom left. Also, $AC = AB$.) Further, AD, the diameter of the circle through O, meets CB at E. If $AC = \sqrt{(15/2)}\,cm$ and $OE = 1\ cm$, estimate the radius of the circle.

46. A ladder $20\ ft$ long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of $3ft/sec$, how fast is the ladder sliding down the building when the top of the ladder is 8 feet from the ground.

47. Find the perimeter of a square with an area same as that of a circle with circumference $30cm$.

48. There are 2 vectors a and b such that $a = < -\sqrt{2}/2, -\sqrt{2}/2 >$ and $b = -6a$. Find the magnitude and direction of b.

49. The maximum safe load for a horizontal beam varies jointly width of the beam and the square of the thickness of the beam and inversely with its length. How will the thickness of the beam have changed if the maximum safe load were halved while the width of the beam was doubled and its length, increased by a factor of 3?

50. An isosceles triangle sits in such a way that neither of its congruent sides is its base. Prove that the bisector of the exterior angle of its top vertex is parallel to its base.

51. What does the expression: $\dfrac{\left(\frac{x^3}{y^3} - 1\right)\sqrt{\left(\frac{x}{y} - 1\right)}^{\,-1}}{\left(\frac{x^2}{y^2} - 1\right)}$ simplify to?

52. A swimming pool, rectangular in shape, is surrounded by a walkway of uniform width, say x. If the outer dimensions of the walkway are $16m$ by $10m$ and the area of the pool alone is $112m^2$, find x.

53. Find all values (or intervals) of x such that a) $y > 1/2$; b) $y = 1/2$; and c) $y < 1/2$ if $y = -sin\, x$ defined over $[-2\pi, 2\pi]$.

54. A rectangle is inscribed inside a semi-circle of diameter $8\ cm$ so that two of its vertices lie symmetrically on the semi-circle with the other two vertices on its diameter. If the area of the rectangle is $\sqrt{7}$, find the dimensions of the rectangle.

55. If $f(x) = x^2 - 7x + k$, find ALL value(s) of k such that $f(x)$ has

   a. No x-intercepts
   b. Exactly 1 x-intercept
   c. 2 x-intercepts