Praxis 5161 Practice Test

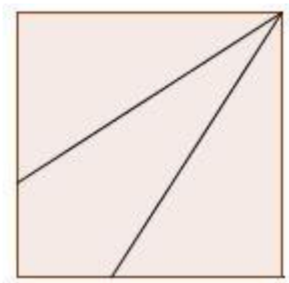
1. Suppose the surface of the earth is smooth and spherical and that the distance around the equator is 25000mi. A steel band is made to fit tightly around the earth at the equator, then the band is cut and a piece of band 18feet long is inserted. What will be the gap, all the way around, between the band and the earth's surface? Assume that 1mi ≈ 5280feet.

Originally, $2\pi r = 25{,}000$ so that $r = 25000/2\pi$; next, $2\pi R = (25{,}000 + 18/5280)$ so that $R = (25{,}000 + 18/5280)/2\pi$. Required, $R - r = (25{,}000 + 18/5280)/2\pi - 25000/2\pi = 0.00054287$mi ≈ 2.8664feet.

2. If n is divided by 5, the remained is 3. What is the remainder when $3n$ is divided by 5?

Since when n is divided by 5, the remained is 3, $n = 5k + 3$, where k is some whole integer. Therefore, $3n = 15k + 9$ ▯ when $15k + 9$ is divided by 5, since $15k$ is divisible by 5, the remainder is 4 when 9 is divided by 5.

3. One corner angle of a square of side 4in is *tri*sected. Find the area of the middle region.



The 2 right triangles are 30°-60°-90° triangles. Since the sides are 4in, the side opposite the 60° angle is $4/\sqrt{3}$, so areas of each right triangle is $\frac{1}{2}\cdot 4/\sqrt{3}\cdot 4$, and the *combined* area is simply $2\cdot\frac{1}{2}\cdot 4/\sqrt{3}\cdot 4 = 16/\sqrt{3}$. Required: $16 - 16/\sqrt{3}\text{in}^2$.

4. It takes a train 45 seconds to pass through a 1320-foot tunnel, and 15 seconds to pass a watchman. How long is the train and what is its speed in miles per hour?

If s is the length of the tunnel and v, the speed of the train, since time = distance / speed, we have $45 = (s + 1320)/v$ and $15 = s/v$. From the latter, $s = 15v$ and substituting this into the 1st, upon cross-multiplying:
$45v = 15v + 1320 \Rightarrow v = 44$feet/second $\approx (44/5280)$ mi / $(1/3600)$ hour = 30mi/hr.

5. The rate of change of an experimental population, A, of fruit-flies is directly proportional to the population at any time, *t*. There were 100 flies on the 2nd day, and 300 on the 4th day. Approximately, how many flies were there in the original population?

Since the rate of change of the population, A, is directly proportional to the population at any time, *t*, i.e. dA/dt = A, it follows the exponential growth model of $A_t = A_o e^{rt}$: we have, $A_o e^{2r} = 100$ and $A_o e^{4r} = 300$, so that dividing the 2nd by the 1st equation, the A_os cancel: $e^{2r} = 3 \Rightarrow r = \frac{1}{2}\ln 3 = 0.5493$. So $A_o e^{2(0.5493)} = 100 \Rightarrow A_o \approx 33$.

6. Suppose the graph of ($\ln x$, y) is linear i.e. using the x-axis as $\ln x$, and the y-axis as y, we get a straight line. If (1, 2) and (e, 3) lie on this line, find its inverse.

Since ($\ln x$, y) is linear, $y = a + b\cdot\ln x$ so that substituting the 2 points:
$2 = a + b\cdot \ln 1 \Rightarrow a = 2$, and $3 = 2 + b\cdot \ln e \Rightarrow b = 1$. Therefore, $y = 2 + \ln x$ is the equation, solving for x: $x = e^{y-2}$.
Therefore, the inverse function is, $y = e^{x-2}$.

7. Calculate Lim $(x \to -⅓)$ $(3 + 7x - 6x^2)/(3x + 1)$

Applying L'Hopital's Rule, since f(-⅓) = 0/0, we get
Lim $(x \to -⅓)$ $(3 + 7x - 6x^2)/(3x + 1)$
≈ Lim $(x \to -⅓)$ $(7 - 12x)/3 = 11/3$, upon taking limits.
NOTE: we could factor $(3 + 7x - 6x^2)$ and solve the problem, too.

8. Determine the Horizontal Asymptotes of $f = \sqrt{(4x^2 - 3x)}/(2x - 5)$

As $x \to \infty$, considering only leading terms: $\sqrt{(4x^2 - 3x)}/(2x - 5) \approx \sqrt{(4x^2)}/(2x) \approx 2x/2x = 1$.
As $x \to -\infty$, considering only leading terms: $\sqrt{(4x^2 - 3x)}/(2x - 5) \approx \sqrt{(4x^2)}/(2x) \approx +2x/-2x = -1$, since the denominator would be a large negative number, whereas the numerator would an identically large albeit positive number!
The H.A. are $y = 1$ and $y = -1$.

9. What value of k shall make $f = (2x^2 - 3kx + 16)/(x - k)$ possess a limit at $x = k$?

Since $x = k$ is a V.A. of $f = (2x^2 - 3kx + 16)/(x - k)$, if f is to possess a limit at $x = k$, then $x - k$ must be a root of $(2x^2 - 3kx + 16)$, so that $2k^2 - 3k \cdot k + 16 = 0 \Rightarrow k = \pm 4$.

10. For $f = \sqrt{(2x - 1)}/(\log x)$, determine intervals of continuity.

$\sqrt{(2x - 1)}$ is defined for $x \geq ½$, and $\log x$ is defined for $x > 0$. Finally, *f* is discontinuous at its V.A., therefore $\log x \neq 0$ $x \neq 1$.

Putting all of this together, *f* is continuous over $(½, 1), (1, \infty)$.

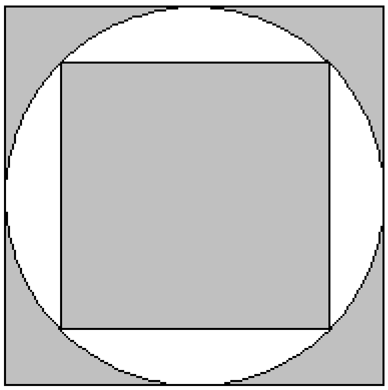
11. The table below gives value of f, f', g and g' for select values of x.

x	-1	1	3	4
f	6	3	1	3
f'	5	-3	-2	-2
g	3	-1	2	5
g'	-2	2	3	2

If $h(x) = f(g(x))$, find $h'(1)$.

Using the Chain Rule, $h'(x) = f'(g(x)) \cdot g'(x)$, so that $h'(1) = f'(g(1)) \cdot g'(1)$ $= f'(-1) \cdot 2 = 5 \cdot 2 = 10$

12. A circle is inscribed in a large square, and in turn, a small square is inscribed in the circle – see below. If a dart were randomly thrown at the figure, what is probability it would land in the region that was white.



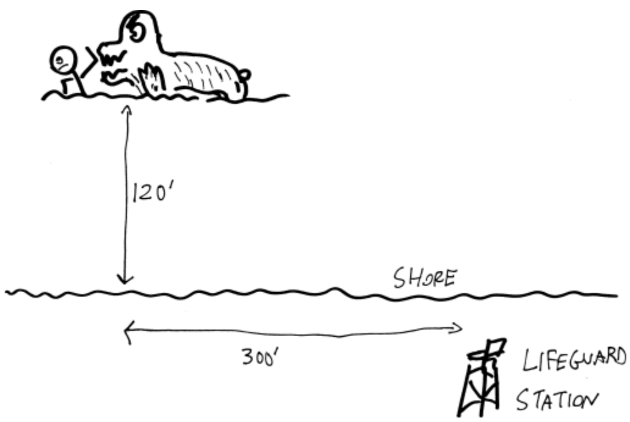
If the side of the large square is s, its area is s^2. Also, the radius of the circle is ½s and its diameter is s…which is the diagonal of the small square, so its side is $s/\sqrt{2}$ and Area is $\frac{1}{2}s^2$. The Area of the circle is $\frac{1}{4}\pi s^2$.

Therefore,

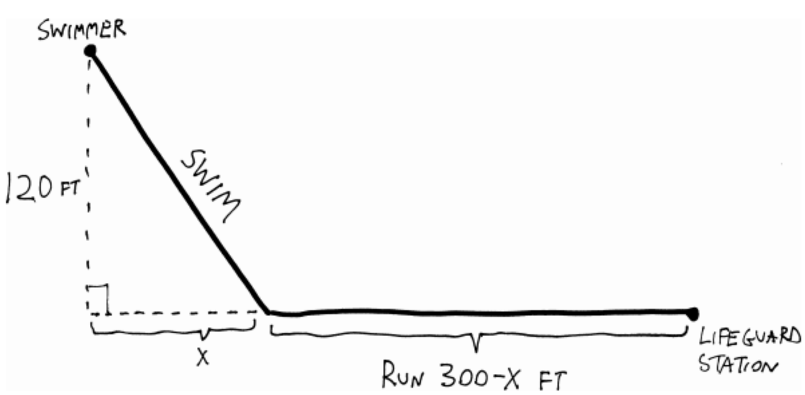
- **the area of the white region is: $(\frac{1}{4}\pi s^2 - \frac{1}{2}s^2)$**
- **the total area is s^2**

so the probability that the dart lands in the white region is, $P = (\frac{1}{4}\pi s^2 - \frac{1}{2}s^2)/s^2 = (\frac{1}{4}\pi - \frac{1}{2}) \approx 0.2854$

13. Suppose a lifeguard at a lifeguard station observes a swimmer about to be attacked by a sea-monster (!). The swimmer is 120ft from the shoreline and the lifeguard is 300feet from *that* point on the shore closest to the swimmer. The lifeguard can run at 13 ft/sec and swim at 5 ft/sec. We wish to minimize the time taken by the lifeguard to reach the swimmer…so, how far down the beach should he run, and how far should he swim? If the lifeguard runs <u>all but</u> the last *x* feet along the shore, express total time taken to reach the swimmer as a function of *x*.



Since the lifeguard runs all but the last x feet along the shore, he runs 300 – x feet…and swims $\sqrt{(120^2 + x^2)}$ feet.



Total time taken, $T(x) = (300 - x)/13 + \sqrt{(120^2 + x^2)}/5$.

14. Find the dimensions of the triangle that must be cut from the corners of a square of side, s, to form a regular octagon.

If x is the side of the isosceles right triangle that must be cut from each corner of the square, its hypotenuse is $\sqrt{2}x$. But this would also be the side of the regular octagon...which is also (s – 2x). Therefore, $s - 2x = \sqrt{2}x$. Solving for x: $x = s/(\sqrt{2} + 2)$.

15. Simplify: $(2^{n+4} - 2\cdot 2^{n}) / 2\cdot 2^{n+3}$

$$\mathbf{(2^{n+4} - 2\cdot 2^{n}) / 2\cdot 2^{n+3} = (2^{n+4} - 2^{n+1}) / 2^{n+4}.}$$

$$\mathbf{= 1 - (2^{n+1})/(2^{n+4}) = 1 - \tfrac{1}{2}^{3} = 7/8.}$$

16. The average age of a group of lawyers and doctors is 40years. If the average age of the doctors is 35, and the average age of the lawyers is 50, what is the ratio of the number of doctors to lawyers?

Let x be the number of lawyers, and y, the number of doctors. Also, the total age of the doctors is 35y, and the total age of the lawyers is 50x. Therefore, the average age of the lawyers and doctors is $(50x + 35y)/(x + y)$. But this is given to be 40. So: $(50x + 35y)/(x + y) = 40$.

Cross-multiplying: $(50x + 35y) = 40(x + y)$ so that
$50x + 35y = 40x + 40y$
** $5y = 10x$**

** $y/x = 2$**

16. Ten ping-pog balls are numbered 1 to 10. If a pair is drawn at random, what sum is most likely to be obtained?

Writing out the possibilities (without replacement) thus: 1, 2; 1, 3; ...; 1, 10, then: 2, 3; 2, 4; ...2, 10, then: 3, 4; 3, 5...3, 10...and so on until 9, 10, we observe that a sum of 11 occurs most number of times (5). Alternately, we could consider all possible sums, starting with 1, 2 = 3 all the way up to 9 + 10 = 19, and consider the different ways those sums could be obtained!

17. For $f = x^2 - 2x - 8$, state the domain of f, $\sqrt{f}$ and $1/f$.

Since f is a parabola, its D is $(-\infty, \infty)$. For $g = \sqrt{x^2 - 2x - 8} = \sqrt{(x - 4)(x + 2)}$, since the expression inside the $\sqrt{\ }$ must be non-negative, using test-values, we get the Domain: $(-\infty, -2), (4, \infty)$. For $h = 1/f = 1/(x^2 - 2x - 8)$, it's V.A. are $x = 4$ and $x = -2$, so the Domain is: $(-\infty, -2), (-2, 4), (4, \infty)$

18. Find the sum of all the numbers that can be obtained by rearranging the digits of the 3-digit number *abc*.

The value of *abc* = 100*a* + 10*b* + c. Since *abc* can be rearranged into *acb, bca, bac, cba* and *cab*...finding each of their values and adding them all up we get: 222*a* + 222*b* 222*c*.

19. A quality control inspector examines DVD players for a complete inspection as they come off an assembly line. It is known that 10% of all items are defective. Also, 60% of all defective items and 20% of all good items undergo a complete inspection. If a randomly selected DVD player undergoes a complete inspection, what is the probability that it is defective?

Given: If D ≈ Defective, and I ≈ Inspection, we have:

1. **$P(D) = 0.1$ □ $P(D') = 0.9$.**
2. **$P(I \mid D) = 0.6$, so using Conditional Probability: $P(I \text{ and } D) = P(I \text{ and } D) \cdot P(D) = 0.6 \cdot 0.1 = 0.06$**
3. **$P(I \mid D') = 0.2$, so using Conditional Probability: $P(I \text{ and } D') = P(I \text{ and } D') \cdot P(D') = 0.2 \cdot 0.9 = 0.18$**

From 2 and 3 above we have, $P(I) = P(I \text{ and } D) + P(I \text{ and } D') = 0.06 + 0.18 = 0.24$.
Required: $P(D \mid I) = P(D \text{ and } I)/P(I) = 0.06/0.24 = 0.25$

20. Find the horizontal and vertical component of the vector with magnitude 8 and direction $5\pi/12$.

The horizontal component is given by $8 \cos 5\pi/12 = 2.0706$, and the vertical component is $8 \sin 5\pi/12 = 7.7274$.

21. If $x^2 + 3xy + y^2 = -1$, what is the slope of the tangent line to the graph at $(2, -1)$?

Differentiating implicitly and using the Product Rule for the middle term: $2x + 3y + 3xy' + 2yy' = 0$. Substituting $(2, -1)$ and solving for y', we get $y' = -\frac{1}{4}$.

22. The IQ scores of the American adult population is roughly normally distributed with a mean of 100 and a standard deviation of 15.
Calculate the IQ that constitutes the 70^{th} percentile for the American population.

If a is the IQ score we need and X denotes IQ scores in general, $P(X \leq a) = 0.7$; using a = InvNorm(0.7, 100, 15) = 107.866.

23. For what intervals of x is $f = x^3 + 3x^2 - 9x + 7$ decreasing <u>and</u> concave up?

For f to be decreasing, $f' = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x + 3)(x - 1) < 0$

For f to be concave up, $f'' = 6x + 6 = 6(x + 1) > 0$.

Using Test-values in the intervals $(-\infty, -3)$, $(-3, -1)$, $(-1, 1)$ and $(1, \infty)$, the required solution is: $(-1, 1)$.