Prerequisites

1. **Master this!** Graph and know characteristics of
   a. $f = \sin x$, $f = \cos x$, $f = \tan x$,
   b. $f = 1/x$, $f = 1/x^2$, $f = 1/x$, $f = 1/x^n$ where n is even, $f = 1/x^m$ where m is odd
   c. $f = ax^2 + bx + c$, $f = a(x - h)^2 + k$, $f = a(x - h)^3 + k$, $f = a\sqrt{(x - h)} + k$,
   d. $f = a(x - h)^{1/3} + k$, $f = a|x - h| + k$, $f = a(x - h)^{2/3} + k$
2. **Master this!** Graph
   a. $f = e^x$, $f = e^{-x}$, $f = -e^x$, $f = -e^{-x}$,
   b. $f = \ln x$, $f = \ln(-x)$, $f = -\ln x$, $f = -\ln(-x)$
3. **Master this!** Graph
   a. $f|x|$: implies $f(-x) = f(x)$ so that the portion of the graph on the *negative* x axis is identical to that on the *positive* x-axis i.e. the left is the reflection of the right.
   b. $|f(x)|$ implies *all* y-values are positive so that the entire negative y portion of the graph is flipped upwards
4. Calculate roots or zeros of functions: on a calculator, solve $f = k$ as the x-Intercepts of $f - k$.
5. **Master this!** Calculate intervals over which $f > 0$ or < 0
6. Calculate V.A. [as $x \to \boldsymbol{a}$, $f \to \pm\infty$] and H.A. [$x \to \pm\infty$, $y \to \boldsymbol{b}$] of functions
7. In general, $a^\infty \to \infty$, if $a > 1$, and $a^\infty \to 0$, if $0 < a < 1$; also, $a/\infty \to 0$ whereas $a/0 \to \infty$.
8. Solve trigonometric equations by hand [$\sin x = -\frac{1}{2}$, $\cos x = -1$, $\tan 2x = 1$], by calculator [$\sin 2x = -\frac{2}{3}$]
9. **Master this!** Calculate trigonometric ratios and inverse-trigonometric values (restricted domain for $\sin^{-1}$ and $\tan^{-1}$ is $[-\pi/2, \pi/2]$ whereas for $\cos^{-1}$ is $[0, \pi]$).
10. Determine if a function is Even, Odd or Neither analytically and graphically
11. Determine Domain and Range of functions analytically and graphically
12. **Master this!** Sketch functions via Domain, Range, Critical Point, x-Intercepts, y-Intercepts, V.A. and H.A.
13. **Master this!** Know *log* rules and Expand and Condense expressions: Product [$\log a{\cdot}b{\cdot}c = \log a + \log b + \log c$] , Quotient [$\log a/b{\cdot}c = \log a - \log b - \log c$] and Power rules [$\log a^b$] = $b{\cdot}\log a$
14. Know $e^{\ln m} = m$ and $\ln e = 1$, $\ln 1 = 0$.
15. Know Rules for Exponents: Multiplication: $x^m{\cdot}x^n = x^{m+n}$; Division: $x^m / x^n = x^{m-n}$; Power: $(x^m)^n = x^{m\cdot n}$
16. Apply the the Number-line trick to solve inequalities and use test-valus when necessary.
17. Rewrite Absolute-value functions as piece-wise functions
18. Find the Sum of an infinite Geometric Series
19. **Master this!** Know trigonometric definitions and identities:
   a. $\sin\theta = y/r$, $\cos\theta = x/r$, $\tan\theta = y/x$,
   b. $\sec\theta = 1/\cos\theta = r/x$, $\text{cosec}\,\theta = 1/\sin\theta = r/y$, $\cot\theta = 1/\tan\theta = x/y$,
   c. $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\tan(-\theta) = -\tan\theta$,
   d. $\cos^2\theta + \sin^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \text{cosec}^2\theta$,
   e. $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 2\sin^2\theta - 1$
20. **Master this!** **DOMAIN NOTES**
   a) Polynomial function **D:** ALL real numbers since there are no breaks
   b) [Even-] Radical function $^e\sqrt{m}$ **D:** set $m \geq 0$ since m must be non-negative
   c) [Odd-] Radical function $^o\sqrt{m}$ **D:** ALL real numbers because we can find the odd-roots of negative numbers!
   d) Rational function, $f = p(x)/q(x)$ **D:** exclude roots of $q(x)$ since $q(x) \neq 0$
   e) Logarithmic functions $\log m$ **D:** set $m > 0$ since log of 0 or negative numbers are undefined

f) Mixed Problems **D:** apply as many rules as they apply...and "merge" the results!
g) **Caution!** For log expressions in the **denominator (**1/log *m***),** observe that log 1 = 0 → insure that ***m*** ≠ 1 by solving ***m*** = 1
h) **Caution!** (x^2 + k) does not have any real solutions i.e. (x^2 + k) is *never* 0.

Limits and Continuity

A. **Master this!** Know the Definition of Limit: LHL = RHL at x = *a*
B. Calculate 1-sided limits
C. **Master this!** Know the Definition of Continuity: LHL = RHL = Value of *f* at x = *a*
D. **Master this!** Know the Definition of Derivative: *f* is continuous at x = *a* **and** L.H.D. = R.H.D. at x = *a*
E. **Master this!** Know the **Intermediate Value Theorem**: If *f* is continuous over [*a, b*] and *f*(*a*) ≤ *N* ≤ *f*(*b*), then there exists at least 1 point *c* in [*a, b*] such that *f*(*c*) = *N*.
F. **Master this!** Know the Special Case of Intermediate Value Theorem ≈ Existence of a root in [*a, b*]: If *f* is continuous over [*a, b*] and *f*(*a*) and *f*(*b*) have opposite signs, then there exists at least 1 point *c* in [*a, b*] such that *f*(*c*) = 0, *a* ≤ *b*.
G. Calculate Limits algebraically: substitute 1st, in case of 0/0 use other methods [factoring, rationalizing, taking LCMs and simplifying, L'Hopital's Rule, using trigonometric identities]
H. Calculate Limits of trigonometric functions [Lim (*m* → 0) sin *m*/*m* = 1, Lim (*m* → 0) tan *m*/*m* = 1] **⟸Master this!**
I. Calculate Limits graphically [examine if the left-hand "height" = right-hand "height" **Caution!** Holes *are* permitted!]
J. Calculate Limits from a table [examine if the left-hand *y*-value = right-hand *y*-value]
K. Calculate Limits of Piece-wise functions: at each "break-point", examine if L.H.L. = R.H.L.
L. Calculate Limits via L'Hopital's Rule [for 0/0 and ∞/∞]
M. Calculate Limits for expressions involving *e* [**I** lim (x → ∞) $(1 + 1/m)^m = e$: in case of **II** lim (x → 0) $(1 + m)^{1/m} = e$ situations *rewrite* to convert to **I,** then evaluate]
N. Calculate Limits for expressions involving 0^∞ or ∞^0 [Take *ln* of both sides, find the limit using L'Hopital's rule…then go to exponential form!]
O. Calculate missing *constants* based on existence of a Limit, Continuity or Differentiability at x = *a, b* [Use definitions of Limit, Continuity and Differentiability]
P. Determine Continuity of Piece-wise functions: at each "break-point" *a,* examine if L.H.L. = R.H.L. = *f*(*a*)
Q. Calculate Limits of *f*unctions as x → ±∞ ≈ Find the H.A. of functions by considering *leading* terms, when relevant, using L'Hopital's Rule, imagining the behavior of known graphs

Derivatives

A. **Master this!** Know the Limit Definition of Derivative at ***a***:
Lim (*h* → 0) f(*a* + *h*) – f(*a*) / *h* **and**
Lim (*x* → *a*) f(x) – f(*a*) / *x* – *a*
B. Recognize and applying terms and notations: average rate of change (m-sec), instantaneous rate of change (m-tan), slope, derivative, slope of the function, slope of the tangent line, *y'*, *dy/dx*
C. Calculate Derivatives of [where *m* is functions of *x*]: m^n, $1/m$, $1/m^n$, $\sqrt{m}$, $\sqrt[n]{m}$, $\sqrt{m^n}$, a^m where *a* is a constant, *a*·*m* where *a* is a constant, *m* / *a* where *a* is a constant, sin *m*, cos *m*, sec *m*, cosec *m*, tan *m*, cot *m*, *ln m*, e^m, $\sin^{-1} m$, $\cos^{-1} m$, $\tan^{-1} m$, $\sec^{-1} m$
D. **Master this!** Calculate Derivatives of Inverse functions: $f^{-1\prime}$ (x) = 1/*f'* ⟸ for this we need *x* that lies on *f* 1st

E. Calculate 1st and 2nd Derivatives Implicitly: use the Chain Rule each time y is derived e.g. $(y)' = y'$, $(y^2)' = 2y \cdot y'$
F. Calculate Derivatives using Product Rule $[(f \cdot g)' = f' \cdot g + f \cdot g']$, Quotient Rule $[(f/g)' = (f' \cdot g - f \cdot g')/g^2)$
G. **Master this!** Apply Chain Rule for Derivatives of Composite functions: $(f(g(x)))' = f'(g(x)) \cdot g'(x))$
H. Calculate Higher Order Derivatives: $y', y'', y''', y^{iv}, \ldots y^{(n)}$ and recognizing patterns
I. Observe that the n-th derivative of a polynomial of degree n, $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_3 x^3 + a_2 x^2 + a_1 x + a_0$ is $n! \cdot a_n$.
J. Observe that the n-th derivative of a polynomial of degree n, $f(x) = (ax + b)^n$ is $n! \cdot a^n$.
K. Write equations of tangent and normal lines
L. Calculate missing constant, k, based on tangents to 2 curves being parallel or perpendicular
M. Calculate points of horizontal [set numerator of $dy/dx = 0$] or vertical tangency [set denominator of $dy/dx = 0$] ⟸**Master this!**
N. Estimating Derivatives at $x = a$ from a table or a graph: use m-sec
O. Determine Differentiability of Piece-wise functions
P. Calculate Derivatives of Absolute-Value functions [rewrite as Piece-wise functions]
Q. Write Linear or Tangent-Line Approximations to f at x: $f(x) = f(a) + f'(a)(x - a)$ and use Linear Approximations to make estimates at x ⟸**Master this!**
R. Recognize that Tangent-Line Approximations to f at x: $f(x) = f(a) + f'(a)(x - a)$ may be an under-estimate or over-estimate based whether f is concave up or concave down

Derivative Formulas

f	f'	f	f'
m/a	$1/a \cdot m'$	$\tan m$	$\sec^2 m \cdot m'$
m^n	$n \cdot m^{n-1} \cdot m'$	$\cot m$	$-\text{cosec}^2\, m \cdot m'$
$1/m$	$-1/m^2 \cdot m'$	$\sec m$	$\sec m \cdot \tan m \cdot m'$
$\sqrt{m}$	$1/2\sqrt{m} \cdot m'$	$\text{cosec}\, m$	$\text{cosec}\, m \cdot \cot m \cdot m'$
a^m	$\ln a \cdot a^m \cdot m'$	$\sin^{-1} m$	$1/\sqrt{(1 - m^2)} \cdot m'$
$\ln m$	$1/m \cdot \cdot m'$	$\cos^{-1} m$	$-1/\sqrt{(1 - m^2)} \cdot m'$
$m^{a/b}$	$a/b \cdot m^{a/b - 1} \cdot m'$	$\tan^{-1} m$	$1/(1 + m^2) \cdot m'$
$\sin m$	$\cos m \cdot m'$	$\log_a m$	$1/(\ln a \cdot m) \cdot m'$
$\cos m$	$-\sin m \cdot m'$		

Application of Derivatives

1. Identify relationships and apply to problems pertaining to related Rates: dx/dt, dy/dt, dA/dt, dC/dt, dV/dt, dS/dt, etc.
2. Recognize situations where f does not have a Limit at $x = a$ [jump, infinite discontinuities]
3. Recognize situations where f is not continuous [point or removable, jump, infinite discontinuities]
4. Recognize situations where f is not differentiable at $x = a$: f has an infinite discontinuity i.e. V.A. at $x = a$ ($f = 1/x$) **OR** f has a vertical tangent at $x = a$ ($f = \sqrt[3]{x}$), **OR** f has sharp corners or a cusp at $x = a$ so that LHD ≠ R.H.D at $x = a$ ($f = |x|$, $f = x^{2/3}$] ⟸**Master this!**
5. Identify intervals over which f is increasing (slope, $f' > 0$) and decreasing (slope, $f' < 0$), using a number line for f'

6. Distinguishing critical points [$f' = 0$ ***or*** d.n.e.] and Extrema [critical point, x where f' switches signs] **⇐Master this!**
7. Recognize that, in special cases: cusps, absolute value functions, the Extrema occurs where f' d.n.e…and the Critical Point ≈ Point of Extrema **⇐Master this!**
8. Identify x where f has a Relative or Local Extrema using the **First Derivative Test**
9. Identify **<u>the</u>** Extrema of f [*y-value* via substitution]
10. Distinguishing *Maxima* [f' changes from + to -] and *Minima* [f' changes from – to +] **⇐Master this!**
11. Identify intervals over which f is concave up / concave down (using a number line for f'')
12. Recognize that when f is concave up, (the slopes are *increasing* $f'' > 0$) and when f is concave down, (the slopes are *decreasing* $f'' < 0$) **⇐Master this!**
13. **CAUTION!** Recognize that slope is positive [suggests f is rising] is DIFFERENT from slope is increasing [suggests f is concave up]
14. Identify Points of Inflection [$f'' = 0$ or d.n.e. and f'' changes signs at $x = a$] **⇐Master this!**
15. Recognize that Points of Inflection are the Extrema of the slopes, y' [slopes are increasing, then decreasing Relative Maxima of slope, y' **OR** slopes are decreasing, then increasing Relative Minima of slope, y'] **⇐Master this!**
16. Identify Increasing and Decreasing intervals from a graph of f'
17. Identify Maxima and Minima from a graph of f'
18. Identify Concave Up and Concave Down intervals from a graph of f'
19. Identify Inflection points from a graph of f'
20. Identify x where f has a Relative or Local Extrema using the **Second Derivative Test** [if a is a critical point such that $f''(a) < 0$, then a is a Relative Maxima; if $f''(a) > 0$, then a is a Relative Minima…whereas if $f''(a) = 0$ or d.n.e. the 2nd Derivative Test is inconclusive…] **⇐Master this!**
21. Apply ideas of Maxima and Minima for solving word-problems relating to maximizing and minimizing Areas, Perimeters, Volumes, Products, Distances, etc.
22. **Master this!** **Existence of Extrema:** If f is continuous over $[a, b]$ then f ***will*** possess, both, an Absolute Maximum and Minimum value in $[a, b]$.
23. **Master this!** **Mean Value Theorem (M.V.T.):** If f is continuous over $[a, b]$ and differentiable over (a, b), then there exists at least 1 point c in (a, b) such that
    a. $f'(c) = f(b) – f(a) / b – a$
    b. the instantaneous rate of change at c = the average rate of change of f over $[a, b]$
    c. m-tan at c = m-sec over $[a, b]$
24. **Master this!** **Rolle's Theorem:** If f is continuous over $[a, b]$, differentiable over (a, b), and $f(a) = f(b)$, then there exists at least 1 point c in (a, b) such that $f'(c) = 0$ [**Psst!** since m-sec = 0!].
25. Recognize that the M.V.T. **fails** if f is **not** continuous over $[a, b]$ OR **not** differentiable over (a, b)
26. Recognize that Rolle's Theorem **fails** if f is **not** continuous over $[a, b]$, **not** differentiable over (a, b) OR $f(a) \neq f(b)$.
27. Sketch graph of f' based on f using ideas pertaining to increasing and decreasing intervals, extrema, concavity and inflection points
28. Sketch graph of f using information pertaining to where f is increasing / decreasing **+** where f is concave up / concave down
29. Identify shape of graphs over intervals where f is increasing + concave down **OR** increasing + concave up **OR** decreasing + concave down **OR** decreasing + concave up **⇐Sketch these NOW!**

30. Identify shape of graphs where $f' > 0, f'' > 0$ **OR** $f' > 0, f'' < 0$ **OR** $f' < 0, f'' > 0$ **OR** $f' < 0, f'' < 0$ **⟸Sketch these NOW!**
31. Identify signs of f' and f'' if f is increasing at an increasing rate **OR** f is increasing at an decreasing rate, **OR** f is decreasing at an increasing rate **OR** f is decreasing at an decreasing rate **⟸Sketch these NOW and identify signs!**
32. Calculate Absolute Extrema of f over $[a, b]$: evaluate f at the Critical points and End-points of the interval **⟸Master this!**
33. **Master this!** Apply ideas pertaining to Extrema and Inflection points for problems relating to motion:
34. Know that $v(t) = s'(t)$ and $a(t) = v'(t) = s''(t)$
35. Know that Velocity function ≈ slope of the position function and acceleration function ≈ slope of the velocity function.
36. Calculate intervals over which object is moving rightwards, leftwards, switches directions
37. Calculate intervals over which the object's velocity is increasing or decreasing or object is at rest
38. Calculate intervals over which the object's acceleration is increasing or decreasing
39. Calculate instantaneous velocity and acceleration at t
40. Calculate average rate of change of position, velocity and acceleration using the definition of Average Rate of Change
41. Calculate maximum or minimum velocity and acceleration over $[t1, t2]$
42. Recognize that the Average Rate of Change of Position, $\Delta s/\Delta t$ = Average Velocity and Average Rate of Change of Velocity, $\Delta v/\Delta t$ = Average Acceleration over $[t1, t2]$
43. If f' does not exist at a, f has an infinite discontinuity i.e. V.A. at $x = a$ ($f = 1/x$) **OR** f has a vertical tangent at $x = a$ ($f = \sqrt[3]{x}$ or $f = \sqrt{x}$), **OR** f has sharp corners or a cusp at $x = a$ so that LHD ≠ R.H.D at $x = a$ [$f = |x|, f = x^{2/3}$] **⟸Master this!**

NOTES on Extrema and Inflection Points

1. ***f*** addresses the ***y*** of the original graph: ***f*** being positive means the graph is above the axis, whereas negative means the graph is below the axis
2. ***f'*** addresses the ***slope*** of the original graph: ***f'*** being positive over an interval means f is increasing over that interval, negative means f is decreasing
3. A **Critical Point** is a point $x = c$ in the Domain of f where $f' = 0$ or **f' d.n.e.**
4. An **Extrema** is a Critical Point ***c*** where ***f'*** changes sign around c i.e. f changes from increasing to decreasing **OR** decreasing to increasing in the interval containing c. It is where ***f*** attains its maximum or minimum i.e. it is the point of highest or lowest f...
5. **CONCEPT**: a Critical Point is (only) a point of ***potential*** Extrema. Not every Critical Point is an Extrema – merely because a point is a Critical Point doesn't automatically make it an Extrema. **Example**, the cube-root $y = x^{1/3}$ has $y' = \frac{1}{3}x^{-2/3} = \frac{1}{3}\cdot 1/x^{2/3}$ so that y' is undefined at $x = 0$ since it has a Vertical Tangent at 0 but ***y*** does not attain an Extrema at $x = 0$!
   Example, the cube $y = x^3$ has $y' = 3x^2$ so that $y' = 0$ at $x = 0$ since it has a Horizontal Tangent at 0 but ***y*** does not attain an Extrema at $x = 0$!
6. **CONCEPT**: **Common Misconception** An **Extrema** is a point ***c*** where **$f' = 0$** or **f' d.n.e.** That is the definition of *Critical Point*. While at every Extrema, **$f' = 0$** or **f' d.n.e** *that* alone is not sufficient to demonstrate that c is an Extrema because not every Critical Point is an Extrema. You still need to check if ***f'*** changes sign around c. That shall determine if c is an Extrema.
7. **CONCEPT**: *Every* Extrema is a Critical Point but not every Critical Point is an Extrema. See **Examples** above: ***f'*** should change signs around the Critical Point, c: f should be increasing, then decreasing around c or vice versa, for it to be an Extrema.

8. **SHORT-CUT:** Examine the x-intercepts of the ***f'*** graph [***f'*** = 0!] and check if a sign-change occurs around each root to determine Extrema
9. ***f ″*** addresses how fast the slope is increasing or decreasing: positive ***f ″*** means the slope ***f '*** is increasing i.e. (*f '*)' **> 0** so that ***f ″*** > 0 ~ Concave Up, whereas negative ***f ″*** means the slope ***f '*** is decreasing i.e. (*f '*)' **< 0** so that ***f ″*** < 0 ~ Concave Down]
10. **DEFINITION** and **CONCEPT** A **Point of Inflection,** *c* occurs where
    a) *f* changes from being Concave Up to Concave Down or Vice Versa
    b) consequently: ***f ″*** changes sign from positive to negative **or** negative to positive: ***f ″*** > 0 and switches to ***f ″*** < 0, **or** vice versa around *c*
    c) alternately: the slope, ***f '*** changes from increasing to decreasing or decreasing to increasing
    d) the slope, ***f '*** attains its maximum or minimum i.e. it is the Extrema of the slope, ***f '*** or it is the point where the slope ***f'*** attains its highest or lowest value…
11. **CONCEPT**: **This is a misconception** "At the point of inflection, ***f ″* = 0** or ***f ″* d.n.e.**
    Um, that is the definition of Critical Point.
    → Now, it **is** true that at every Point of Inflection, ***f″* = 0** or ***f ″* d.n.e** but *that* alone is <u>not</u> sufficient to demonstrate that *c* is an Inflection Point. You still need to check if ***f ″*** changes sign around *c*. <u>That</u> shall determine if *c* is a Point of Inflection. **NOTE:** If there is insufficient information to demonstrate a sign-change, then work with *f″* = 0.
12. **SHORT-CUT:**
    a) Examine the *f* graph and observe where it is Concave Up and where, Concave Down: the Point where the *switch* occurs is the Point of Inflection.
    b) Examine the *f′* graph and observe where *f '* is increasing [slope is increasing: Concave Up] and *f '* is decreasing [slope is decreasing: Concave Down]: the Point where the switch occurs is the Point of Inflection.
    Alternately, it is the Relative Maximum or Minimum of the *f′* graph!
    c) Examine the *f″* graph and if it switches sign at its x-intercepts: the roots where the switch occurs is the Point of Inflection.
13. **Method 2 for Relative Extrema**
    a) Since a Relative Minima occurs where *f* is Concave Up, if ***c*** is a Critical Point [***f '* = 0** or ***f '* d.n.e**], then ***f ″***(***c***) > 0
    b) Since a Relative Maxima occurs where *f* is Concave Down, if ***c*** is a Critical Point [***f'* = 0** or ***f '* d.n.e**], then ***f ″***(***c***) < 0.

Similarities and Differences: Extrema vs. Inflection Points

1. Extrema relates to Max or Min values of the ***f*** curve whereas Point of Inflection relates to Max or Min values of the ***f'*** curve.
2. Extrema occur at the x-intercepts of the ***f*** 'curve [***f'*** = 0] whereas Point of Inflections occur at the Max or Min values of the ***f '*** curve **OR** at the x-intercepts of the ***f*** ″curve [***f″*** = 0].
3. When ***f'*** > 0, the original function *f* is increasing whereas when ***f″*** > 0, the slope ***f'*** is increasing [(***f'***)***'*** > 0].
4. To check for Extrema, we examine if a sign change occurs at *c* for ***f'*** whereas to check for Point of Inflection, we examine if a sign change occurs at *c* for ***f ″***.

NOTES on Motion

1. An object on the right if its position, s(t) > 0, on the left if s(t) < 0.
2. An object is moving **rightwards** if v(t) > 0, on the **leftwards** if v(t) < 0.
3. An object maybe on the left of the Origin moving rightwards or on the right of the Origin, moving leftwards: **position** and **direction of motion** are 2 different aspects!

4. An object can ***potentially*** change directions when v(t) = 0: a sign change must occur.
5. Velocity is the rate of change of distance. Velocity could be the average velocity [m-sec] or instantaneous [m-tan].
6. Average velocity = $\Delta s / \Delta t$
7. Instantaneous velocity at time, *t*, v(t) = s'(t)
8. Acceleration is the rate of change of velocity. Acceleration could be the average acceleration [m-sec] or instantaneous [m-tan].
9. Average acceleration = $\Delta v/\Delta t$
10. Instantaneous acceleration, a(t) = v'(t)
11. An object's speed is increasing **OR** is speeding up when *v*(t) and *a*(t) have the same sign.
12. An object's speed is decreasing **OR** is slowing down when *v*(t) and *a*(t) have opposite signs.
13. The signs of velocity and acceleration have to do with ***direction***…they are unrelated to magnitude. An object moving right may have the exact velocity as one moving left…likewise, an object moving right may have the exact acceleration as one moving left!
14. An object's distance, from the origin, x(t), is increasing when x(t) > 0 and x'(t) > 0 **OR** x(t) < 0 and x'(t) < 0.

Integration

1. Know the Limit Definition of Riemann Sum: A = Lim ($n \to \infty$) $\sum$(i =1 to n) f(x_i) Δx where Δx = $(b - a)/n$ **⟸Master this!**
2. Recognize that if *f* is the derivative of *g* *f* = *g'*, then *g* is the anti-derivative or integral of *f* *g* = ∫*f* dx + C **⟸Master this IDEA / notation!**
   [Also, taking integrals of both sides of *g'* = *f*, we get *g* = ∫*f* dx + C]
3. Recognize that if *f* is the anti-derivative or integral of *g* *f* = ∫*g* dx, then *g* is the derivative of *f* *g* = *f'* **⟸Master this IDEA / notation!**
   [Also, taking derivatives of both sides of *f* = ∫*g* dx, we get *f'* = *g*]
4. Calculate integrals of $(ax + b)^n$, $1/(ax + b)$, $^n\sqrt{(ax + b)^m}$, $(ax + b)^{m/n}$, $e^{(ax + b)}$, sin ($ax + b$), cos ($ax + b$), $\sec^2(ax + b)$, $\csc^2(ax + b)$, $\sec(ax + b)\tan(ax + b)$, $\csc(ax + b)\cot(ax + b)$, $1/\sqrt{(1 - x^2)}$, $1/(1 + x^2)$, $1/x\sqrt{(1 - x^2)}$ **⟸Make a formula sheet NOW!**
5. **Master this!** Know Properties of Integrals:
   a) ∫ *kf* ± *g* dx = *k*∫*f* dx ± ∫*g* dx
   b) ∫(*a* to *b*) *f* dx = -∫(*b* to *a*)*f* dx
   c) ∫(*a* to *b*) *f* dx + ∫(*b* to *c*) *f* dx = ∫(*a* to *c*)*f* dx
   d) ∫(*a* to *a*)*f* dx = 0
   e) If $f \leq g$ over [*a, b*], then ∫(*a* to *b*) *f* dx ≤ ∫(*a* to *b*)*g* dx
   f) If $m \leq f \leq M$ over [*a, b*], then $m(b - a) \leq$ ∫(*a* to *b*) *f* dx ≤ $M(b - a)$
6. **Master this!** **FTC I:** If F(x) = ∫*f* dx, then ∫(*a* to *b*) *f* dx = F(*b*) – F(*a*)
7. **Master this!** **Net Change Theorem**:
   a) **Forwards:** ∫(*a* to *b*) *f'* dx = *f*(*b*) – *f*(*a*)
   b) **Backwards:** *f*(*b*) – *f*(*a*) = ∫(*a* to *b*) *f'* dx
   c) **Variation-Forwards:** ∫(*a* to *b*) *f''* dx = *f'*(*b*) – *f'*(*a*)
   d) **Variation-Backwards:** *f'*(*b*) – *f'*(*a*) = ∫(*a* to *b*) *f''* dx
8. **Master this!** **Net Change Theorem Applications:**
   a) ∫(*t*1 to *t*2) *v*(t) *dt* = ∫(*t*1 to *t*2) *s'*(t) *dt* = *s*(*t*2) – *s*(*t*1)
   b) ∫(*t*1 to *t*2) *a*(t) *dt* = ∫(*t*1 to *t*2) *v'*(t) *dt* = *v*(*t*2) – *v*(*t*1)
   c) In general, ∫(*a* to *b*) *P'*(t) *dt* = *P*(*b*) – *P*(*a*)
9. **Master this!** **Net Change Theorem Corollary**: *f*(x) = *f*(*a*) + ∫(*a* to *x*) *f'* dx

10. **Master this! Net Change Theorem Corollary Applications**:
$s(t2) = s(t1) + \int(t1 \text{ to } t2)\, s'(t)\, dt = s(t1) + \int(t1 \text{ to } t2)\, v(t)\, dt$
$v(t2) = v(t1) + \int(t1 \text{ to } t2)\, v'(t)\, dt = v(t1) + \int(t1 \text{ to } t2)\, a(t)\, dt$
11. Calculate / Estimating Areas [Distance / Displacement] via Left Riemann Sum, Right Riemann Sum, Trapezoidal Riemann Sum and Mid-Point Riemann Sum.
12. Creating a table of values *x vs. f*(x) to compute Left Riemann Sum, Right Riemann Sum, Trapezoidal Riemann Sum and Mid-Point Riemann Sum
13. Distinguishing between Total Distance Traveled [*ignore* the negative signs in the velocity function] and Displacement [consider the *negative* signs in the velocity function]
14. **Master this!** Considering the intercepts of the velocity function and splitting the integral up suitably when calculating Total Distance: $s = |\int(t1 \text{ to } t2)\, v(t)\, dt| + |\int(t2 \text{ to } t3)\, v(t)\, dt| + \ldots$
15. Using u-substitution to evaluate integrals and balancing expressions carefully
16. **Master this!** Using **Change of Variables + Change of Limits** when using u-substitution to compute Definite Integrals
17. **Master this!** Distinguishing between
    a) $\int a\, dx/(b + cx^2)$: use $\tan^{-1}$ after suitable adjustments
    b) $\int ax^2\sqrt{(b \pm cx)}\, dx$: use $u = (b - cx)$ and substitute for *x* suitably in the numerator
    c) $\int ax\, dx/(b \pm cx^2)$: use $u = (b + cx^2)$
    d) $\int a\, dx/(b - cx^2)$: use Partial fractions by rewriting the denominator as Difference of Squares after suitable adjustments
    e) $\int ax\, dx/\sqrt{(b \pm cx^2)}$: use $u = (b - cx^2)$
    f) $\int ax^2/\sqrt{(bx \pm c)}\, dx$: use $u = (bx - c)$ and substitute for *x* suitably in the numerator
    g) $\int a\, dx/(b \pm cx)$: use *ln*
    h) $\int a\, dx/\sqrt{(b \pm cx)}$: use Power Rule after suitable adjustments
    i) $\int a\, dx/\sqrt{(b - cx^2)}$: use $\sin^{-1}$ after suitable adjustments
18. **Master this!** Calculate Average Value of a function, *f* over [*a, b*]: $1/(b - a)\cdot\int f\, dx$
19. **Master this!** Calculate Integrals of Piece-wise functions [split up the integral suitably at the limits 1st!]
20. Calculate Integrals of Absolute-value functions [find the x-intercepts 1st and split up the integral suitably, adjusting the limits accordingly]
21. Account for the initial value condition in Qs: e.g. V(0) = *m,* s(0) = *d*, P(0) = *p*…etc. and incorporate that information in accumulation or integral computation at other points e.g. V(*t*), s(t), P(t)…etc.
22. **Master this!** Using Geometry to calculate Definite Integrals: be observant about signs & direction:
    a) Proceeding *left to right* values of $\int (a \text{ to } b)\, f\, dx$ are POSITIVE for $f > 0$
    b) Proceeding *left to right* values of $\int (a \text{ to } b)\, f\, dx$ are NEGATIVE for $f < 0$
    c) Proceeding *right to left* values of $\int (a \text{ to } b)\, f\, dx$ are NEGATIVE for $f > 0$
    d) Proceeding *right to left* values of $\int (a \text{ to } b)\, f\, dx$ are POSITIVE for $f < 0$
23. **CAUTION!** Do not confuse Average (value) of a function [$\mathbf{1/(b - a)\cdot\int f\, dx}$]with Average Rate of Change of a function [$\mathbf{f(b) - f(a)/b - a}$]
24. **Master this! FTC II:** The *derivative* of $\int(g \text{ to } h)\, f\, dx = f(h)\cdot h' - f(g)\cdot g'$
25. Computing the value of the Accumulation function, $F(x) = \int(a \text{ to } x)\, f\, dx$ for different values of *a*
26. Find the 1st and 2nd Derivative of the Accumulation function, $F(x) = \int(a \text{ to } x)\, f\, dx$ via FTC II
27. Calculate Increasing and Decreasing intervals, Extrema, intervals of concavity and Inflection Points for the Accumulation function, $F(x) = \int(a \text{ to } x)\, f\, dx$ analytically and graphically
28. Recognize relationships between Average Rate of Change and Average Value:

a. the average rate of change of velocity over [t1, t2] is **m-sec**
= v(t2) – v(t1) / (t2 – t1)...which may ALSO be written as
= ∫ (t1 to t2) *v'*(t) dt/ (t2 – t1) using the Net Change Theorem...and since *v'* = *a*(t)
= ∫ (t1 to t2) *a*(t) dt/ (t2 – t1)...which is the average acceleration over [t1, t2]!

b. the average velocity over [t1, t2] is
∫ (t1 to t2) *v*(t) dt/ (t2 – t1)
= ∫ (t1 to t2) *s'*(t) dt/ (t2 – t1)
= s(t2) – s(t1) / (t2 – t1) [using the Net Change Theorem...]
...which is the average rate of change of distance!

c. the average rate of change of distance over [t1, t2] is **m-sec**
= s(t2) – s(t1) / (t2 – t1)...which may ALSO be written as
= ∫ (t1 to t2) *s'*(t) dt/ (t2 – t1) using the Net Change Theorem...and since *s'* = *v*(t)
= ∫ (t1 to t2) *v*(t) dt/ (t2 – t1)...which is the average velocity over [t1, t2]!

d. the average acceleration over [t1, t2] is
∫ (t2 to t1) *a*(t) dt/ (t2 – t1)
= ∫ (t2 to t1) *v'*(t) dt/ (t2 – t1)
= v(t2) – v(t1) / (t2 – t1) [using the Net Change Theorem...]
...which is the average rate of change of velocity!

29. Calculate integrals of composite functions or transformations upon *f*: For ∫*f*(*a*x + *b*) dx **OR** ∫*f*(*g*(x)) dx, try u-substitution u = (*a*x + *b*) **OR** u = *g*(x) and adjust limits suitably, if necessary.

f	**∫*f* dx**	***f***	**∫*f* dx**
$(ax + b)^n$	$(ax + b)^{n+1}/[a \cdot (n + 1)]$	$\sec^2(ax + b)$	$\tan(ax + b)/a$
$1/(ax + b)$	$\ln(ax + b)/a$	$\csc^2(ax + b)$	$-\cot(ax + b)/a$
$e^{(ax + b)}$	$e^{(ax + b)}/a$	$\sec(ax + b)\cdot\tan(ax + b)$	$\sec(ax + b)/a$
$\sqrt{(ax + b)}$	$⅔(ax + b)^{3/2}/a$	$\csc(ax + b)\cdot\cot(ax + b)$	$-\csc(ax + b)/a$
$1/\sqrt{(ax + b)}$	$2\sqrt{(ax + b)}/a$	$1/\sqrt{(1 - x^2)}$	$\sin^{-1}x$
$\sin(ax + b)$	$-\cos(ax + b)/a$	$1/(1 + x^2)$	$\tan^{-1}x$
$\cos(ax + b)$	$\sin(ax + b)/a$	$1/x\sqrt{(1 - x^2)}$	$\sec^{-1}x$
		a^x	$a^x/\ln a$

Integration Methods

1. **Master this!** Identify **Slope Fields**:
   a) Consider combinations of (x, y) that make the slope, *dy/dx* = 0, 1, -1, undefined
   b) Consider how the slope, *dy/dx* changes [increases or decreases, remains constant?] as *x* increases / decreases [moving ⇄] and *y* increases / decreases [moving ⇅].
   c) Consider the *sign* of the slopes in the 4 quadramts
2. **Master this!** Evaluating Integrals *via* Integration by Parts **(BC)**: ∫*f*·*g'* dx = *f*·*g* – ∫*f'*·*g* dx
   Tip! Choose *f* and *g'* so that *f* is easy to differentiate and *g'* is easy to integrate
3. **Master this!** Evaluating Integrals *via* Integration by Partial Fractions **(BC)**: By factoring the denominator into 2 linear expressions, decompose $\int(Cx + D)/(ax^2 + bx + c) = \int P/(Mx + N)\,dx + \int Q/(Ox + P)\,dx$
4. **Master this!** Evaluating Improper Integrals **(BC)**: An integral with definite integral limits of [*a, b*] is improper when
   a) **Case I** one / both limits [*a, b*] is ±∞: evaluate integral, apply limits at ±∞
   b) **Case II** one or both limits of the Definite integral is a V.A.: evaluate integral, apply 1-sided limits at *a* and / or *b*

c) **Case III** an infinite discontinuity lies within [*a*, *b*]: split up the integral at the discontinuity *c*, and take limits

Differential Equations

1. Solve Differential Equations via Separation of Variables: ***first***, rewrite *f '* as *dy/dx*.
2. **Master this!** Setting up Differential Equations for word problems:
    a. Rate of Change of *y* is proportional to *y*: $dy/dt = ky$
    b. Rate of Change of *y* is inversely proportional to $\sqrt{y}$: $dy/dt = k/\sqrt{y}$
    c. Rate of Change of *y* is proportional to $(N - y)$: $dy/dt = k(N - y)$
    d. Rate of Change of *y* is proportional to product of *y* and $(N - y)$: $dy/dt = ky(N - y)$
3. **Master this!** Recognize that the formulation "Rate of Change of *y* is proportional to *y*: $dy/dt = ky$" yields the Exponential model: $At = Aoe^{rt}$
4. Solve word problems relating to Exponential growth and decay: population growth, radio-active decay
5. **Master this!** Apply ideas pertaining to Anti-derivatives for motion-related problems
    a. **Forwards:** ∫(*t*1 to *t*2) *v*(t) *dt* = ∫(*t*1 to *t*2) *s'*(t) *dt* = *s*(*t*2) – *s*(*t*1)
    Backwards: *s*(*t*2) – *s*(*t*1) = ∫(*t*1 to *t*2) *s'*(t) *dt* = ∫(*t*1 to *t*2) *v*(t) *dt*
    b. **Forwards:** ∫(*t*1 to *t*2) *a*(t) *dt* = ∫(*t*1 to *t*2) *v'*(t) *dt* = *v*(*t*2) – *v*(*t*1)
    Backwards: *v*(*t*2) – *v*(*t*1)= ∫(*t*1 to *t*2) *v'*(t) *dt* = ∫(*t*1 to *t*2) *a*(t) *dt*
    c. *s*(*t*2) = *s*(*t*1) + ∫(*t*1 to *t*2) *s'*(t) dt = *s*(*t*1) + ∫(*t*1 to *t*2) *v*(t) dt
    d. *v*(*t*2) = *v*(*t*1) + ∫(*t*1 to *t*2) *v'*(t) dt = *v*(*t*1) + ∫(*t*1 to *t*2) *a*(t) dt
6. **Master this!** Apply Euler's Method **(BC):** Given (xo, yo) and dy/dx, using the tangent-line approximation, estimate $y1 = y0 + y'0(x1 - x0)$, and successively as $y2 = y1 + y'1(x2 - x1)$ and so on.
7. **Master this!** Know Properties of Logistic Functions **(BC)**:
    a. The General Differential Equation is $dy/dt = ky(1 - y/L)$ where *k* is the constant of proportionality, and *L* is the Carrying Capacity ≈ H.A.
    b. Alternately: $dy/dt = ky - ky^2/L$
    c. The general solution is $y = L/(1 + be^{-kt})$ and the Point of Inflection is $y = ½L$.
    d. The Logistic function is *S*-shaped with y-Intercept of $L/(1 + b)$ and H.A.: $y = 0$ and $y = L$.

Applications of Integration

1. Find Area bounded between *f* and the x-axis –
    a. Find the x-intercepts of *f*
    b. Set up the integral [**splitting up the integral limits as necessary**]: A = ∫(x1 to x2) *f* dx + ∫(x3 to x4) *f* dx + …
2. **Master this!** Find Area bounded between *f* and *g* –
    a. Find the points of intersection of *f* and *g*
    b. Set up the integral as [**splitting up the integral limits as necessary**]:
    A = ∫(x1 to x2) (*f* – *g*) dx + ∫(x3 to x4) (*g* – *f*) dx + …
    NOTE: In general, in the x-direction, we perform Top function – Bottom function, and in the *y*-direction, Right – Left function.
3. Find Volume of solid obtained by generating region bounded by *f* and the x-axis –
    a. Find the x-intercepts of *f*
    b. Set up the integral as [**splitting up the integral as necessary**]:
    V = **π**∫(x1 to x2) f^2 dx + **π**∫(x3 to x4) f^2 dx + …

4. **Master this!** Find Volume of solid obtained by generating region bounded by f and g about the x-axis –
   a. Find the points of intersection of f and g
   b. Set up the integral as [**splitting up the integral as necessary**]:
      V = **π**∫(x1 to x2) ($f^2 - g^2$) dx + **π** ∫(x3 to x4) ($g^2 - f^2$) dx + …
      NOTE: In general, we perform Outer Radius^2 – Inner Radius^2
5. **Master this!** Find Volume of solid whose base lies along bounded region, R, and whose cross-section perpendicular to the x-axis is a
   a. Square: V = ∫ (x1 to x2) $\boldsymbol{f^2}$ **dx**
   b. Isosceles Right Triangle: V = ½∫(x1 to x2) $\boldsymbol{f^2}$ **dx**
   c. Equilateral Triangle: V = $\boldsymbol{\sqrt{3}/4}$∫ (x1 to x2) $\boldsymbol{f^2}$ **dx**
   d. Semi-circle: V = ⅛π ∫ (x1 to x2) $\boldsymbol{f^2}$ **dx**
   e. Rectangle of height g: V = ∫(x1 to x2) $\boldsymbol{f \cdot g}$ **dx**
6. **Master this!** Find the lengths of Arcs **(BC)**:
   a. For function, f between a and b: l = ∫√(1 + f'^2)dx
   b. For parametric function, $x = f(t)$ and $y = g(t)$ between $t1$ and $t2$: l = ∫√[(x'(t)^2 + $y'(t)^2$)] dt

Polar, Parametric and Vector Functions (BC)

1. **Parametric Functions**
   a. Find the derivative of Parametric Functions at t: $x = f(t)$ and $y = g(t)$ via $dy/dx = dy/dt / dx/dt$.
   b. **Master this!** Find the 2nd derivative for Parametric functions: $d^2y/dx^2 = dy'/dt / dx/dt$.
   c. Find points of Horizontal tangency: set $dy/dt = 0$ and points of Vertical tangency: set $dx/dt = 0$.
   d. **Master this!** Find the length of the arc: if $x = f(t)$, $y = g(t)$ the length, between $t1$ and $t2$:
      l = ∫√[(x'(t)^2 + $y'(t)^2$)] dt
2. **Polar Functions:**
   a. **Master this!** $r = f(\theta)$ is the distance from the Origin (0, 0) Find the rate of change of the distance from the origin via $dr/dt = f'(\theta) \cdot d\theta/dt$
   b. **Master this!** Identify Polar functions $r = f(\theta)$ quickly:
      i. $r = a \cos\theta$ **and** $r = a \sin\theta$ are circles.
      ii. $r = a + a \cos\theta$ **and** $r = a + a \sin\theta$ are cardiods
      iii. $r = a + b \cos\theta$ **and** $r = a + b \sin\theta$, $a > b$ are "out limacons"
      iv. $r = a + b \cos\theta$ **and** $r = a + b \sin\theta$, $a < b$ are "in limacons": there's a loop inside
      v. $r = a \cos 2\theta$, $r = \sin 2\theta$, $r = a \cos 3\theta$, $r = \sin 3\theta$ are rose-petals.
   c. **Master this!** Graph Polar functions quickly by finding r for the quadrant angles: 0°, 90°, 180°, 270° and 360°
      CAUTION! Observe *where* 0°, 90°, 180°, 270° and 360° lie. The 2nd quadrant does **not** necessarily mean [π/2, π]. It depends on what r was at θ = 0 and π/2 and π.
   d. Find r and the x- and y- coordinates corresponding to θ via $r = f(\theta)$ and $x = r \cos\theta$, $y = r \sin\theta$
   e. Find the x- and y- coordinates, given r : find θ 1st and use $x = r \cos\theta$, $y = r \sin\theta$
   f. **Master this!** Find slope of the tangent line to $r = f(\theta)$ at θ: use $x = r \cos\theta$, $y = r \sin\theta$ and find $dy/dx = dy/d\theta / dx/d\theta$.
   g. Find the 2nd derivative for Polar functions: $d^2y/dx^2 = dy'/d\theta / dx/d\theta$.

h. Determine Increasing and Decreasing intervals, and Interbals of Concavity using ***y'*** and ***y''***.
i. **Master this!** Find points of Horizontal tangency: set $dy/d\theta = 0$ and points of Vertical tangency: set $dx/d\theta = 0$.
j. **Master this!** Find the Area bounded by r between θ1 and θ2, $A = ½ \int r^2\, d\theta$
NOTE: Account for symmetry…
k. **Master this!** Find the Area bounded by $r1$ and $r2$ between θ1 and θ2, $A = ½ \int (r1^2 - r2^2)\, d\theta$
NOTE: First, find the points of intersection θ1 and θ2…
l. For interpretation of $dr/d\theta$ at a given θ, consider the **sign** of r and $dr/d\theta$. If BOTH have the same signs, then r is increasing! r is the **distance** of the object from the **origin.**
For interpretation of $dx/d\theta$ at a given θ, consider the **sign** of x and $dx/d\theta$. If BOTH have the same signs, then x is increasing. x is the **distance** of the object from the **y-axis!**
For interpretation of $dy/d\theta$ at a given θ, consider the **sign** of y and $dy/d\theta$. If BOTH have the same signs, then y is increasing. y is the **distance** of the object from the **x-axis!**

3. **Vector-Valued functions**
   a. **Master this!** Find the Position Function: **(**x = x(t), y = y(t)**)** at time t if v(t) is given using Net Change Theorem Corollary:
   $x(t1) = x(t0) + \int(t0 \text{ to } t1)\, v(t)\, dt$ and
   $y(t1) = y(t0) + \int(t0 \text{ to } t1)\, v(t)\, dt$
   b. **Master this!** Find the Velocity Function: $V(t) = \langle x'(t) = dx/dt,\ y'(t) = dy/dt \rangle$
   c. **Master this!** Find the Acceleration Function: $\boldsymbol{a}(t) = v'(t) = \langle x''(t) = d^2x/dt^2,\ y''(t) = d^2y/dt^2 \rangle$.
   d. **Master this!** Find the Speed of the object: Speed is the Magnitude of the Velocity vector. Since the magnitude (or "length") of *any* vector, $\mathbf{u} = \langle x, y \rangle = x\boldsymbol{i} + y\boldsymbol{j}$ is given by $|\mathbf{u}| = \sqrt{x^2 + y^2}$…Speed $= |\mathbf{v}| = \sqrt{[x'(t)^2 + y'(t)^2]} = \sqrt{(dx/dt)^2 + (dy/dt)^2}$
   e. **Master this!** Find the Distance traveled by the particle or object, $d \approx$ Length of the Arc, $l = \int\sqrt{[x'(t)^2 + y'(t)^2]}\, dt = \int\sqrt{[(dx/dt)^2 + (dy/dt)^2}\, dt$…using the Length of Arc formula from Parametric functions.
   f. The slope of the tangent line to the position curve is $dy/dx = dy/dt\ /\ dx/dt$.

Sequences and Series (BC)

1. **Master this! Determine the Convergence of Sequences:**
   a. In case of Convergence of sequence, $\{\mathbf{a}n\}$ converges if $\text{Lim}\ (n \to \infty)\ \mathbf{a}n$ exists. The Limit could be zero or ***any*** real number.
   b. For sequences, for $\text{Lim}\ (n \to \infty)\ \mathbf{a}n$
      i. consider *leading terms* of expressions (in the numerator and denominator) and then apply the limit
      ii. Use L'Hopital's Rule in case of ∞/∞ [Take the derivative and apply the limit as $n \to \infty$ *repeatedly*]
      iii. If $|r| < 1$, $\lim (n \to \infty)\ r^n = 0$. Because a $fraction^\infty \approx 0$, if $|fraction| < 1$
      iv. If $|r| > 1$, $\lim (n \to \infty)\ r^n$ ***d.n.e.*** Because a $fraction^\infty \approx \infty$, if $|fraction| > 1$.
      v. → The factorial function, **n!** grows much faster than the exponential function, $\boldsymbol{a}^n$.

vi. → The polynomial function, n^a grows much slower than the exponential function, a^n.
vii. → The polynomial function, n^a grows much faster than the logarithmic function, $\ln n$.

2. **Master this! Determine the Convergence of SERIES Guidelines:**
   a. a) Apply the n-th term test to check for *divergence*. **Note:** The n-th term Test does *not* determine convergence, ZOMG!
   b. b) Is the series one of the special types: Geometric, *p*-series, telescoping?
   c. c) Is a_n easily integrable (Integral Test)? Can the Ratio Test be applied?
   d. d) Can the series be **compared** to one of the special types of series? [See *b*]

3. **Master this! Apply the n-th term Test** *Always*, perform the **n-th term Test**:
   a. For the n-th term, if $\lim (n \to \infty)\ a_n \neq 0$, the Series diverges!
   b. For the n-th term, if $\lim (n \to \infty)\ a_n = 0$, the Series *could* converge (or diverge) → proceed to one of the OTHER tests.
   Caution! The n-th term test ***cannot*** be used to determine convergence, *only* divergence. That the *n*-th term of a series is 0 is a necessary but **not** sufficient condition for convergence!

4. **Apply the Telescoping Series, Geometric Series and p-series Tests** Determine if the series is
   a. Telescoping $[1/(an^2 + bn + c)]$: A Telescoping Series always converges: use Partial Fractions to write $[1/(cn + d)(en + f)] \approx 1/(cn + d) - 1/(en + f)$, expand and resolve!

   b. **Master this!** Geometric [underlying exponential function: $k \cdot b^n$ ← **n is the exponent**]:
      i. A Geometric Series converges if $|r| < 1$.
      ii. **SPECIAL NOTE**: Complicated series of the form $\sum(1 \text{ to } \infty)\ 2^n \cdot (-3)^{n+1}/7^{n-1}$ can be rewritten as: $\sum(1 \text{ to } \infty)\ \mathbf{2} \cdot 2^{n-1} \cdot \mathbf{(-1)^2} \cdot (-1)^{n-1} \cdot \mathbf{3^2} \cdot (3)^{n-1}/7^{n-1}$ $= \mathbf{18} \sum(1 \text{ to } \infty)\ (-6/7)^{n-1}$ which is a geometric series with $a = 18$ and $|r| = 6/7 < 1$!
      iii. Find the Sum of an Infinite Geometric Series: $\sum a \cdot r^n = a/(1 - r)$

   c. **Master this!** *p*-series [underlying polynomial function: $\mathbf{1}/n^b$ ← **n is the base**]:
      i. A *p*-series converges if $p > 1$.
      ii. Complicated series of the form $\sum(1 \text{ to } \infty)\ \sqrt{n}/\sqrt[3]{n^5}$ can be rewritten as: $\sum(1 \text{ to } \infty)\ 1/n^{-½} \cdot n^{5/3} = \sum(1 \text{ to } \infty)\ 1/n^{7/3}$ which is a *p*-series with $p = 7/3 > 1$!

5. **Master this! Apply the Integral Test** In case $a_n \approx f(x)$ is positive, continuous and decreasing, then $\sum(k \text{ to } \infty)\ a_n \approx \int(k \text{ to } \infty)\ \mathbf{f(x)\,dx}$ converge or diverge together.

6. **Master this! Apply the Comparison Tests**
   a. **Direct Comparison Test**: Suppose that we have two series $\sum a_n$ and $\sum b_n$ with a_n and $b_n \geq 0$ for all n and $a_n \leq b_n$ for all n. Then,
      i. If $\sum b_n$ is convergent then so is $\sum a_n$.
      ii. If $\sum a_n$ is divergent then so is $\sum b_n$.

7. **Limit Comparison Test:** Suppose that we have two series $\sum \mathbf{a}n$ and $\sum \mathbf{b}n$ with $\mathbf{a}n$ and $\mathbf{b}n \geq 0$ for all *n.* Then, ***if*** lim (n → ∞) $\mathbf{a}n / \mathbf{b}n$ exists and is non-zero, then $\sum \mathbf{a}n$ and $\sum \mathbf{b}n$ converge or diverge together.

   NOTE: Choose b**n** such that lim ($n \to \infty$) a**n** ≈ b**n** **OR** use your knowledge of the behaviours of *known* series: factorial function, n! >> exponential function, a^n >> polynomial function, n^a >> logarithmic function, log n

8. **Master this! Apply the Ratio Test**: If $\sum \mathbf{a}n$ is a series with non-zero terms. Then,
   a. $\sum \mathbf{a}n$ converges if lim ($n \to \infty$) $|\mathbf{a}_{n+1} / \mathbf{a}_n| < 1$,
   b. $\sum \mathbf{a}n$ diverges if lim ($n \to \infty$) $|\mathbf{a}_{n+1} / \mathbf{a}_n| > 1$ or $= \infty$
   c. The Ratio Test is inconclusive if lim ($n \to \infty$) $|\mathbf{a}_{n+1} / \mathbf{a}_n| = 1$.

9. **Master this! Apply the n*th*-Root Test**: If $\sum \mathbf{a}n$ is a series with non-zero terms. Then,
   a. $\sum \mathbf{a}n$ converges if lim ($n \to \infty$) $(|\mathbf{a}_n|)^{1/n} < 1$,
   b. $\sum \mathbf{a}n$ diverges if lim ($n \to \infty$) $(|\mathbf{a}_n|)^{1/n} > 1$ or $= \infty$
   c. The n*th*-Root Test is inconclusive if lim ($n \to \infty$) $(\mathbf{a}_n)^{1/n} = 1$.
      NOTE: You <u>should</u> take the absolute value of $\mathbf{a}_n$ i.e. $|\mathbf{a}_n|$ *before* applying the n*th*-Root Test.

10. **Master this! Apply Tests for Alternating Series** An Alternating Series, $\sum \mathbf{a}n$ [involving $(-1)^n$] converges **IF**
    a. **Absolute Convergence Test** $\sum |\mathbf{a}n|$ converges **← Super-fast approach! OR**
    b. **Alternate Series Test** Lim ($n \to \infty$) $|\mathbf{a}n| = 0$ **and** $\mathbf{a}_{n+1} \leq a_n$. For this, show that $a_n - \mathbf{a}_{n+1} \geq 0$ **OR** $\mathbf{a}_{n+1} / \mathbf{a}_n > 1$ **OR** if $\mathbf{a}n \approx f(x)$, then $\boldsymbol{f}'(x) < 0$.
    c. An Alternating Series, $\sum \mathbf{a}n$ [involving $(-1)^n$] such that $\sum |\mathbf{a}n|$ diverges but $\sum \mathbf{a}n$ converges is said to be **Conditionally Convergent.** E.g. $\sum (-1)^n (1/n)$, $\sum (-1)^n (1/\sqrt{n})$

Master this! Radius of Convergence and Interval of Convergence

Step 1: Apply the Ratio Test: If $\sum \mathbf{a}n$ is a series with non-zero terms, then

- $\sum \mathbf{a}n$ converges if lim ($n \to \infty$) $|\mathbf{a}_{n+1} / \mathbf{a}_n| < 1$,
- $\sum \mathbf{a}n$ diverges if lim ($n \to \infty$) $|\mathbf{a}_{n+1} / \mathbf{a}_n| > 1$ or $= \infty$
- The Ratio Test is inconclusive if lim ($n \to \infty$) $|\mathbf{a}_{n+1} / \mathbf{a}_n| = 1$.

Simplify the expression in | | as much as possible and take limits.

Step 2: Determine the Radius of Convergence. For this, for the simplified expression in **Step 1** in the form lim ($n \to \infty$) $|(x - b)/k|$

NOTE: there *may* or may not be a b i.e. b may be 0…

NOTE: there *may* or may not be a k i.e. k may be 1…

- **Case I:** If the expression lim ($n \to \infty$) $|g(n) \cdot (x - b)/k| \to \infty > 1$ [this shall happen if there's *still* an n term in the numerator, after simplification…], then $\sum \mathbf{a}n$ diverges for all x except b,
  - the Radius of Convergence, R = 0 **← Grasp / Memorize This!**
  - the Interval of Convergence, I.O.C. is x = {b} [the root of the expression $|(x - b)|$…] **← Grasp / Memorize This!**
- **Case II:** If the expression lim ($n \to \infty$) $|(x - b)/ g(n) \cdot k| \to 0 < 1$ [this shall happen if there's *still* an n term in the denominator, after simplification…], then $\sum \mathbf{a}n$ converges for *all* x,

- the Radius of Convergence, $R = \infty$ ← **Grasp / Memorize This!**
- the Interval of Convergence, I.O.C. is $(-\infty, +\infty)$ ← **Grasp / Memorize This!**

- **Case III:** If the expression $\lim (n \to \infty) \mid g(n)\cdot(x - b)/k\mid$ is neither 0 nor ∞ and is a function of x [this shall happen if $\lim (n \to \infty) \mid g(n) = 1$], then, according to the Ratio Test: $\sum$**a***n* converges if $|(x - b)/k| < 1$, cross-multiplying, we get:
  $|x - b| < k$ ⟸ **This** is the Radius of Convergence, **R.**

Step 3: Determine the Interval of Convergence by solving Absolute Value Inequality: $|x - b| < k$
$-k < (x - b) < k$
$-k + b < x < k + b$ ⟸ **This is the "Preliminary" Interval of Convergence**

Step 4: Determine the Convergence of the Series *at* the 2 end-points of the interval:
$x = -k + b$ and
$x = k + b$

For this substitute these values of x **carefully** into the ORIGINAL series $\sum$**a***n* and examine the convergence of $\sum$**a***n* at both end-points using one of the Convergence Tests learned earlier! This shall yield one of the following possibilities:

a) $-k + b < x < k + b$ [if $\sum$**a***n* diverges at BOTH end-points!]
b) $-k + b \leq x < k + b$ [if $\sum$**a***n* diverges at the right end-point!]
c) $-k + b < x \leq k + b$ [if $\sum$**a***n* diverges at the left end-point!]
d) $-k + b \leq x \leq k + b$ [if $\sum$**a***n* converges at BOTH end-points!]

11. **Master this!** Find the **Remainder or Error of an Alternating Series**: For an alternating series whose terms decrease in absolute terms to 0, the error in using n terms to approximate the series, $|S - S_n| \leq a_{n+1}$, the 1st omitted or skipped term.

12. **Master this!** Computing **Maclaurin Series:** The Maclaurin series about x = 0 is,
$f(x) = f(0) + f'(0)/1!\cdot x + f''(0)/2!\cdot x^2 + f'''(0)/3!\cdot x^3 + f^{(iv)}(0)/4!\cdot x^4 + \ldots f^{(n)}(0)/n!\cdot x^n + \ldots$
Here, the coefficients or constants, depicted by $f(0)/0!, f'(0)/1!, f''(0)/2!, f'''(0)/3!, f^{(iv)}(0)/4! \ldots f^{(n)}(0)/n!$ are also denoted by $a_0, a_1, a_2, a_3, a_4 \ldots a_n \ldots$respectively.

In general, the coefficients of x^n are: $a_n = f^{(n)}(0)/n!$ ⟸**Master this!**

The n-th derivative of the n-th degree Maclaurin series about x = 0 is $f^{(n)}(0)$.

13. **Master this!** Computing **Taylor Series:** The Taylor series about x = c is:
$f(x) = f(c) + f'(c)/1!\cdot(x - c) + f''(c)/2!\cdot(x - c)^2 + f'''(c)/3!\cdot(x - c)^3 + f^{(iv)}(c)/4!\cdot(x - c)^4 + \ldots f^{(n)}(c)/n!\cdot(x - c)^n + \ldots$

so that the coefficients or constants, depicted by $f(c), f'(c)/1!, f''(c)/2!, f'''(c)/3!, f^{(iv)}(c)/4!, \ldots f^{(n)}(c)/n!$ are also denoted by $a_0, a_1, a_2, a_3, a_4 \ldots a_n \ldots$respectively.

In general, then, the coefficients of $(x - c)^n$ are: $a_n = f^{(n)}(c)/n!$ ⟸**Master this!**

14. **Memorize this!** Apply Important Power Series –
a) $1/(1 - x) = 1 + x + x^2 + \ldots x^n + \ldots$
b) $\sin x = x - x^3/3! + x^5/5! - \ldots + (-1)^n\cdot x^{2n-1}/2n - 1! + \ldots$

c) $\cos x = 1 - x^2/2! + x^4/4! - \ldots + (-1)^n \cdot x^{2n}/2n! + \ldots$
d) $e^x = 1 + x/1! + x^2/2! + x^3/3! + \ldots x^n/n! + \ldots$
e) $ln\,(1 + x) = x - x^2/2 + x^3/3 - \ldots + (-1)^{n+1} \cdot x^{n+1}/n + 1 + \ldots$
f) $\tan^{-1} x = x - x^3/3 + x^5/5 - \ldots + (-1)^n \cdot x^{2n+1}/2n + 1 + \ldots$

15. Use *known* Series
a) to determine Limits, Derivatives, Integrals and Composite functions e.g. $f(x^2)$: write 1st *n* terms and general term
b) to calculate Radius of Convergence and Interval of Convergence for Derivatives, Integrals and Composite functions of given Series
c) to demonstrate that given series of *y* satisfies an equation: $g(y, y'', \ldots) = 0$ via careful substitution and combining Like terms

16. **Special Note:** Estimating $f(x)$: Given, $(a, f(a))$ we may estimate $f(x)$
a) Using Derivatives ≈ Tangent Line Approximation: $f(x) = f(a) + f'(a)(x - a)$
b) Using Integrals ≈ Net Change Theorem Corollary: $f(x) = f(a) + \int(a \text{ to } x) f'(x)\, dx$
c) Using Polynomial Approximation ≈ Maclaurin Series: $\boldsymbol{f}(x) = \boldsymbol{f}(0) + \boldsymbol{f}'(0)/1! \cdot x + \boldsymbol{f}''(0)/2! \cdot x^2 + \boldsymbol{f}'''(0)/3! \cdot x^3 + \boldsymbol{f}^{(iv)}(0)/4! \cdot x^4 + \ldots \boldsymbol{f}^{(n)}(0)/n! \cdot x^n + \ldots$